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Over the last 40 years, a revolution has occurred in the application of electric motors. The development of solid-state motor drive packages has progressed to the point where practically any power control problem can be solved by using them. With such solid-state drives, it is possible to run dc motors from ac power supplies or ac motors from dc power supplies. It is even possible to change ac power at one frequency to ac power at another frequency.

Furthermore, the costs of solid-state drive systems have decreased dramatically, while their reliability has increased. The versatility and the relatively low cost of solid-state controls and drives have resulted in many new applications for ac motors in which they are doing jobs formerly done by dc machines. DC motors have also gained flexibility from the application of solid-state drives.

This major change has resulted from the development and improvement of a series of high-power solid-state devices. Although the detailed study of such power electronic circuits and components would require a book in itself, some familiarity with them is important to an understanding of modern motor applications.

This chapter is a brief introduction to high-power electronic components and to the circuits in which they are employed. It is placed at this point in the book because the material contained in it is used in the discussions of both ac motor controllers and dc motor controllers.

3.1 POWER ELECTRONIC COMPONENTS

Several major types of semiconductor devices are used in motor-control circuits. Among the more important are
1. The diode
2. The two-wire thyristor (or PNPN diode)
3. The three-wire thyristor [or silicon controlled rectifier (SCR)]
4. The gate turnoff (GTO) thyristor
5. The DIAC
6. The TRIAC
7. The power transistor (PTR)
8. The insulated-gate bipolar transistor (IGBT)

Circuits containing these eight devices are studied in this chapter. Before the circuits are examined, though, it is necessary to understand what each device does.

The Diode

A diode is a semiconductor device designed to conduct current in one direction only. The symbol for this device is shown in Figure 3–1. A diode is designed to conduct current from its anode to its cathode, but not in the opposite direction.

The voltage-current characteristic of a diode is shown in Figure 3–2. When a voltage is applied to the diode in the forward direction, a large current flow results. When a voltage is applied to the diode in the reverse direction, the current flow is limited to a very small value (on the order of microamperes or less). If a large enough reverse voltage is applied to the diode, eventually the diode will break down and allow current to flow in the reverse direction. These three regions of diode operation are shown on the characteristic in Figure 3–2.

Diodes are rated by the amount of power they can safely dissipate and by the maximum reverse voltage that they can take before breaking down. The power

\[ i_D \]
\[ v_D \]

FIGURE 3–1
The symbol of a diode.

\[ i_D \]
\[ v_D \]

FIGURE 3–2
Voltage-current characteristic of a diode.
dissipated by a diode during forward operation is equal to the forward voltage drop across the diode times the current flowing through it. This power must be limited to protect the diode from overheating. The maximum reverse voltage of a diode is known as its peak inverse voltage (PIV). It must be high enough to ensure that the diode does not break down in a circuit and conduct in the reverse direction.

Diodes are also rated by their switching time, that is, by the time it takes to go from the off state to the on state, and vice versa. Because power diodes are large, high-power devices with a lot of stored charge in their junctions, they switch states much more slowly than the diodes found in electronic circuits. Essentially all power diodes can switch states fast enough to be used as rectifiers in 50- or 60-Hz circuits. However, some applications such as pulse-width modulation (PWM) can require power diodes to switch states at rates higher than 10,000 Hz. For these very fast switching applications, special diodes called fast-recovery high-speed diodes are employed.

The Two-Wire Thyristor or PNPN Diode

Thyristor is the generic name given to a family of semiconductor devices which are made up of four semiconductor layers. One member of this family is the two-wire thyristor, also known as the PNPN diode or trigger diode. This device's name in the Institute of Electrical and Electronics Engineers (IEEE) standard for graphic symbols is reverse-blocking diode-type thyristor. Its symbol is shown in Figure 3–3.

The PNPN diode is a rectifier or diode with an unusual voltage-current characteristic in the forward-biased region. Its voltage-current characteristic is shown in Figure 3–4. The characteristic curve consists of three regions:

1. The reverse-blocking region
2. The forward-blocking region
3. The conducting region

In the reverse-blocking region, the PNPN diode behaves as an ordinary diode and blocks all current flow until the reverse breakdown voltage is reached. In the conducting region, the PNPN diode again behaves as an ordinary diode, allowing large amounts of current to flow with very little voltage drop. It is the forward-blocking region that distinguishes a PNPN diode from an ordinary diode.
When a PNPN diode is forward-biased, no current flows until the forward voltage drop exceeds a certain value called the breakover voltage $V_{BO}$. When the forward voltage across the PNPN diode exceeds $V_{BO}$, the PNPN diode turns on and remains on until the current flowing through it falls below a certain minimum value (typically a few milliamperes). If the current is reduced to a value below this minimum value (called the holding current $I_H$), the PNPN diode turns off and will not conduct until the forward voltage drop again exceeds $V_{BO}$.

In summary, a PNPN diode

1. Turns on when the applied voltage $v_D$ exceeds $V_{BO}$
2. Turns off when the current $i_D$ drops below $I_H$
3. Blocks all current flow in the reverse direction until the maximum reverse voltage is exceeded

The Three-Wire Thyristor or SCR

The most important member of the thyristor family is the three-wire thyristor, also known as the silicon controlled rectifier or SCR. This device was developed and given the name SCR by the General Electric Company in 1958. The name thyristor was adopted later by the International Electrotechnical Commission (IEC). The symbol for a three-wire thyristor or SCR is shown in Figure 3–5.
As the name suggests, the SCR is a controlled rectifier or diode. Its voltage-current characteristic with the gate lead open is the same as that of a PNPN diode.

What makes an SCR especially useful in motor-control applications is that the breakover or turn-on voltage of the device can be adjusted by a current flowing into its gate lead. The larger the gate current, the lower $V_{BO}$ becomes (see Figure 3–6). If an SCR is chosen so that its breakover voltage with no gate signal is larger than the highest voltage in the circuit, then it can only be turned on by the application of a gate current. Once it is on, the device stays on until its current falls below $I_H$. Therefore, once an SCR is triggered, its gate current may be removed without affecting the on state of the device. In the on state, the forward voltage drop across the SCR is about 1.2 to 1.5 times larger than the voltage drop across an ordinary forward-biased diode.

Three-wire thyristors or SCRs are by far the most common devices used in power-control circuits. They are widely used for switching or rectification applications and are currently available in ratings ranging from a few amperes up to a maximum of about 3000 A.

In summary, an SCR

1. Turns on when the voltage $v_D$ applied to it exceeds $V_{BO}$
2. Has a breakover voltage $V_{BO}$ whose level is controlled by the amount of gate current $i_G$ present in the SCR
3. Turns off when the current $i_D$ flowing through it drops below $I_H$
4. Blocks all current flow in the reverse direction until the maximum reverse voltage is exceeded

The Gate Turnoff Thyristor

Among the recent improvements to the thyristor is the gate turnoff (GTO) thyristor. A GTO thyristor is an SCR that can be turned off by a large enough negative pulse at its gate lead even if the current $i_D$ exceeds $I_H$. Although GTO thyristors have been around since the 1960s, they only became practical for motor-control applications in the late 1970s. Such devices are becoming more and more common in motor-control packages, since they eliminate the need for external components to turn off SCRs in dc circuits (see Section 3.5). The symbol for a GTO thyristor is shown in Figure 3–7a.

Figure 3–7b shows a typical gate current waveform for a high-power GTO thyristor. A GTO thyristor typically requires a larger gate current for turn-on than an ordinary SCR. For large high-power devices, gate currents on the order of 10 A or more are necessary. To turn off the device, a large negative current pulse of 20- to 30-μs duration is required. The magnitude of the negative current pulse must be one-fourth to one-sixth that of the current flowing through the device.
The DIAC

A DIAC is a device containing five semiconductor layers (PNPNP) that behaves like two PNPN diodes connected back to back. It can conduct in either direction once the breakover voltage is exceeded. The symbol for a DIAC is shown in Figure 3–8, and its current-voltage characteristic is shown in Figure 3–9. It turns on when the applied voltage in either direction exceeds $V_{BO}$. Once it is turned on, a DIAC remains on until its current falls below $I_H$.

The TRIAC

A TRIAC is a device that behaves like two SCRs connected back to back with a common gate lead. It can conduct in either direction once its breakover voltage
is exceeded. The symbol for a TRIAC is shown in Figure 3–10, and its current-voltage characteristic is shown in Figure 3–11. The breakover voltage in a TRIAC decreases with increasing gate current in just the same manner as it does in an SCR, except that a TRIAC responds to either positive or negative pulses at its gate. Once it is turned on, a TRIAC remains on until its current falls below $I_H$.

Because a single TRIAC can conduct in both directions, it can replace a more complex pair of back-to-back SCRs in many ac control circuits. However, TRIACs generally switch more slowly than SCRs, and are available only at lower power ratings. As a result, their use is largely restricted to low- to medium-power applications in 50- or 60-Hz circuits, such as simple lighting circuits.
The Power Transistor

The symbol for a transistor is shown in Figure 3–12a, and the collector-to-emitter voltage versus collector current characteristic for the device is shown in Figure 3–12b. As can be seen from the characteristic in Figure 3–12b, the transistor is a device whose collector current $i_C$ is directly proportional to its base current $i_B$ over a very wide range of collector-to-emitter voltages ($v_{CE}$).

Power transistors (PTRs) are commonly used in machinery-control applications to switch a current on or off. A transistor with a resistive load is shown in Figure 3–13a, and its $i_C-v_{CE}$ characteristic is shown in Figure 3–13b with the load line of the resistive load. Transistors are normally used in machinery-control applications as switches; as such they should be either completely on or completely off. As shown in Figure 3–13b, a base current of $i_{B4}$ would completely turn on this transistor, and a base current of zero would completely turn off the transistor.

If the base current of this transistor were equal to $i_{B3}$, then the transistor would be neither fully on nor fully off. This is a very undesirable condition, since a large collector current will flow across a large collector-to-emitter voltage $v_{CE}$, dissipating a lot of power in the transistor. To ensure that the transistor conducts without wasting a lot of power, it is necessary to have a base current high enough to completely saturate it.

Power transistors are most often used in inverter circuits. Their major drawback in switching applications is that large power transistors are relatively slow in changing from the on to the off state and vice versa, since a relatively large base current has to be applied or removed when they are turned on or off.
The Insulated-Gate Bipolar Transistor

The insulated-gate bipolar transistor (IGBT) is a relatively recent development. It is similar to the power transistor, except that it is controlled by the voltage applied to a gate rather than the current flowing into the base as in the power transistor. The impedance of the control gate is very high in an IGBT, so the amount of current flowing in the gate is extremely small. The device is essentially equivalent to the combination of a metal-oxide-semiconductor field-effect transistor (MOSFET) and a power transistor. The symbol of an IGBT is shown in Figure 3–14.
Since the IGBT is controlled by a gate voltage with very little current flow, it can switch much more rapidly than a conventional power transistor can. IGBTs are therefore being used in high-power high-frequency applications.

**Power and Speed Comparison of Power Electronic Components**

Figure 3–15 shows a comparison of the relative speeds and power-handling capabilities of SCRs, GTO thyristors, and power transistors. Clearly SCRs are capable of higher-power operation than any of the other devices. GTO thyristors can operate at almost as high a power and much faster than SCRs. Finally, power transistors can handle less power than either type of thyristor, but they can switch more than 10 times faster.

![Figure 3-15](image-url)  
**FIGURE 3–15**  
A comparison of the relative speeds and power-handling capabilities of SCRs, GTO thyristors, and power transistors.
3.2 BASIC RECTIFIER CIRCUITS

A rectifier circuit is a circuit that converts ac power to dc power. There are many different rectifier circuits which produce varying degrees of smoothing in their dc output. The four most common rectifier circuits are

1. The half-wave rectifier
2. The full-wave bridge rectifier
3. The three-phase half-wave rectifier
4. The three-phase full-wave rectifier

A good measure of the smoothness of the dc voltage out of a rectifier circuit is the ripple factor of the dc output. The percentage of ripple in a dc power supply is defined as the ratio of the rms value of the ac components in the supply's voltage to the dc value of the voltage

$$ r = \frac{V_{ac,\text{rms}}}{V_{DC}} \times 100\% $$  \hspace{1cm} (3-1)

where $V_{ac,\text{rms}}$ is the rms value of the ac components of the output voltage and $V_{DC}$ is the dc component of voltage in the output. The smaller the ripple factor in a power supply, the smoother the resulting dc waveform.

The dc component of the output voltage $V_{DC}$ is quite easy to calculate, since it is just the average of the output voltage of the rectifier:

$$ V_{DC} = \frac{1}{T} \int v(t) \, dt $$  \hspace{1cm} (3-2)

The rms value of the ac part of the output voltage is harder to calculate, though, since the dc component of the voltage must be subtracted first. However, the ripple factor $r$ can be calculated from a different but equivalent formula which does not require the rms value of the ac component of the voltage. This formula for ripple is

$$ r = \left( \frac{V_{rms}}{V_{DC}} \right)^2 - 1 \times 100\% $$  \hspace{1cm} (3-3)

where $V_{rms}$ is the rms value of the total output voltage from the rectifier and $V_{DC}$ is the dc or average output voltage from the rectifier.

In the following discussion of rectifier circuits, the input ac frequency is assumed to be 60 Hz.

The Half-Wave Rectifier

A half-wave rectifier is shown in Figure 3–16a, and its output is shown in Figure 3–16b. The diode conducts on the positive half-cycle and blocks current flow on
the negative half-cycle. A simple half-wave rectifier of this sort is an extremely poor approximation to a constant dc waveform—it contains ac frequency components at 60 Hz and all its harmonics. A half-wave rectifier such as the one shown has a ripple factor $r = 121$ percent, which means it has more ac voltage components in its output than dc voltage components. Clearly, the half-wave rectifier is a very poor way to produce a dc voltage from an ac source.

Example 3-1. Calculate the ripple factor for the half-wave rectifier shown in Figure 3-16, both analytically and using MATLAB.

**Solution**

In Figure 3-16, the ac source voltage is $v_s(t) = V_M \sin \omega t$ volts. The output voltage of the rectifier is

$$v_{load}(t) = \begin{cases} V_M \sin \omega t & 0 < \omega t < \pi \\ 0 & \pi \leq \omega t \leq 2\pi \end{cases}$$

Both the average voltage and the rms voltage must be calculated in order to calculate the ripple factor analytically. The average voltage out of the rectifier is

$$V_{DC} = V_{avg} = \frac{1}{T} \int_0^T v_{load}(t) \, dt = \frac{\omega}{2\pi} \int_0^{\pi/\omega} V_M \sin \omega t \, dt$$

$$= \frac{\omega}{2\pi} \left[ -\frac{V_M}{\omega} \cos \omega t \right]_0^{\pi/\omega}$$

FIGURE 3-16

(a) A half-wave rectifier circuit.

(b) The output voltage of the rectifier circuit.
The rms value of the total voltage out of the rectifier is

\[ V_{\text{rms}} = \sqrt{\frac{1}{T} \int_{0}^{T} V_{\text{load}}(t)^2 \, dt} \]

\[ = \sqrt{\frac{2}{\pi} \int_{0}^{\pi/\omega} V_{M}^2 \sin^2 \omega t \, dt} \]

\[ = V_{M} \sqrt{\frac{2}{\pi} \int_{0}^{\pi/\omega} \frac{1}{2} \cos 2\omega t \, dt} \]

\[ = V_{M} \sqrt{\frac{2}{\pi} \int_{0}^{\pi/\omega} \frac{1}{2} dt - \frac{\omega}{2\pi} \int_{0}^{\pi/\omega} \frac{1}{2} \cos 2\omega t \, dt} \]

\[ = V_{M} \sqrt{\frac{2}{\pi} \left[ \frac{1}{4} - \frac{1}{8\pi} \sin 2\pi \right] - \left( 0 - \frac{1}{8\pi} \sin 0 \right)} \]

\[ = \frac{V_{M}}{2} \]

Therefore, the ripple factor of this rectifier circuit is

\[ r = \frac{\sqrt{\frac{V_{M}/2}{V_{M}/\pi}}}{1 \times 100\%} = 121\% \]

The ripple factor can be calculated with MATLAB by implementing the average and rms voltage calculations in a MATLAB function, and then calculating the ripple from Equation (3–3). The first part of the function shown below calculates the average of an input waveform, while the second part of the function calculates the rms value of the input waveform. Finally, the ripple factor is calculated directly from Equation (3–3).

```matlab
function r = ripple(waveform)
% Function to calculate the ripple on an input waveform.

% Calculate the average value of the waveform
nvals = size(waveform,2);
temp = 0;
for ii = 1:nvals
    temp = temp + waveform(ii);
end
average = temp/nvals;

% Calculate rms value of waveform
temp = 0;
```
for ii = 1:nvals
    temp = temp + waveform(ii)^2;
end
rms = sqrt(temp/nvals);

% Calculate ripple factor
r = sqrt((rms/average)^2 - 1) * 100;

Function ripple can be tested by writing an m-file to create a half-wave rectified waveform and supply that waveform to the function. The appropriate M-file is shown below:

% M-file: test_halfwave.m
% M-file to calculate the ripple on the output of a half-wave % wave rectifier.

% First, generate the output of a half-wave rectifier
waveform = zeros(1,128);
for ii = 1:128
    waveform(ii) = halfwave(ii*pi/64);
end

% Now calculate the ripple factor
r = ripple(waveform);

% Print out the result
string = ['The ripple is ' num2str(r) '%.'];
disp(string);

The output of the half-wave rectifier is simulated by function halfwave.

function volts = halfwave(wt)
% Function to simulate the output of a half-wave rectifier.
% wt = Phase in radians (=omega x time)

% Convert input to the range 0 <= wt < 2*pi
while wt >= 2*pi
    wt = wt - 2*pi;
end
while wt < 0
    wt = wt + 2*pi;
end

% Simulate the output of the half-wave rectifier
if wt >= 0 & wt <= pi
    volts = sin(wt);
else
    volts = 0;
end

When test_halfwave is executed, the results are:

> test_halfwave
The ripple is 121.1772%.

This answer agrees with the analytic solution calculated above.
The Full-Wave Rectifier

A full-wave bridge rectifier circuit is shown in Figure 3-17a, and its output voltage is shown in Figure 3-17c. In this circuit, diodes $D_1$ and $D_3$ conduct on the positive half-cycle of the ac input, and diodes $D_2$ and $D_4$ conduct on the negative half-cycle. The output voltage from this circuit is smoother than the output voltage from the half-wave rectifier, but it still contains ac frequency components at 120 Hz and its harmonics. The ripple factor of a full-wave rectifier of this sort is $r = 48.2$ percent—it is clearly much better than that of a half-wave circuit.

![Figure 3-17](image)

(a) A full-wave bridge rectifier circuit. (b) The output voltage of the rectifier circuit. (c) An alternative full-wave rectifier circuit using two diodes and a center-tapped transformer.
Another possible full-wave rectifier circuit is shown in Figure 3–17b. In this circuit, diode $D_1$ conducts on the positive half-cycle of the ac input with the current returning through the center tap of the transformer, and diode $D_2$ conducts on the negative half-cycle of the ac input with the current returning through the center tap of the transformer. The output waveform is identical to the one shown in Figure 3–17c.

The Three-Phase Half-Wave Rectifier

A three-phase half-wave rectifier is shown in Figure 3–18a. The effect of having three diodes with their cathodes connected to a common point is that at any instant the diode with the largest voltage applied to it will conduct, and the other two diodes will be reverse-biased. The three phase voltages applied to the rectifier circuit are shown in Figure 3–18b, and the resulting output voltage is shown in Figure 3–18c. Notice that the voltage at the output of the rectifier at any time is just the highest of the three input voltages at that moment.
This output voltage is even smoother than that of a full-wave bridge rectifier circuit. It contains ac voltage components at 180 Hz and its harmonics. The ripple factor for a rectifier of this sort is 18.3 percent.

The Three-Phase Full-Wave Rectifier

A three-phase full-wave rectifier is shown in Figure 3–19a. Basically, a circuit of this sort can be divided into two component parts. One part of the circuit looks just like the three-phase half-wave rectifier in Figure 3–18, and it serves to connect the highest of the three phase voltages at any given instant to the load.

The other part of the circuit consists of three diodes oriented with their anodes connected to the load and their cathodes connected to the supply voltages (Figure 3–19b). This arrangement connects the lowest of the three supply voltages to the load at any given time.

Therefore, the three-phase full-wave rectifier at all times connects the highest of the three voltages to one end of the load and always connects the lowest of the three voltages to the other end of the load. The result of such a connection is shown in Figure 3–20.
The output of a three-phase full-wave rectifier is even smoother than the output of a three-phase half-wave rectifier. The lowest ac frequency component present in it is 360 Hz, and the ripple factor is only 4.2 percent.

Filtering Rectifier Output

The output of any of these rectifier circuits may be further smoothed by the use of low-pass filters to remove more of the ac frequency components from the output. Two types of elements are commonly used to smooth the rectifier’s output:

1. Capacitors connected across the lines to smooth ac voltage changes
2. Inductors connected in series with the line to smooth ac current changes

A common filter in rectifier circuits used with machines is a single series inductor, or choke. A three-phase full-wave rectifier with a choke filter is shown in Figure 3–21.
3.3 PULSE CIRCUITS

The SCRs, GTO thyristors, and TRIACs described in Section 3.1 are turned on by the application of a pulse of current to their gating circuits. To build power controllers, it is necessary to provide some method of producing and applying pulses to the gates of these devices at the proper time to turn them on. (In addition, it is necessary to provide some method of producing and applying negative pulses to the gates of GTO thyristors at the proper time to turn them off.)

Many techniques are available to produce voltage and current pulses. They may be divided into two broad categories: analog and digital. Analog pulse generation circuits have been used since the earliest days of solid-state machinery controls. They typically rely on devices such as PNPN diodes that have voltage-current characteristics with discrete nonconducting and conducting regions. The transition from the nonconducting to the conducting region of the device (or vice versa) is used to generate a voltage and current pulse. Some simple analog pulse generation circuits are described in this section. These circuits are collectively known as relaxation oscillators.

Digital pulse generation circuits are becoming very common in modern solid-state motor drives. They typically contain a microcomputer that executes a program stored in read-only memory (ROM). The computer program may consider many different inputs in deciding the proper time to generate firing pulses. For example, it may consider the desired speed of the motor, the actual speed of the motor, the rate at which it is accelerating or decelerating, and any specified voltage or current limits in determining the time to generate the firing pulses. The inputs that it considers and the relative weighting applied to those inputs can usually be changed by setting switches on the microcomputer’s circuit board, making solid-state motor drives with digital pulse generation circuits very flexible. A typical digital pulse generation circuit board from a pulse-width-modulated induction motor drive is shown in Figure 3–22. Examples of solid-state ac and dc motor drives containing such digital firing circuits are described in Chapters 7 and 9, respectively.

The production of pulses for triggering SCRs, GTOs, and TRIACs is one of the most complex aspects of solid-state power control. The simple analog circuits

---

**FIGURE 3–21**
A three-phase full-wave bridge circuit with an inductive filter for reducing output ripple.
A typical digital pulse generation circuit board from a pulse-width-modulated (PWM) induction motor drive. (Courtesy of MagneTek Drives and Systems.)

![Figure 3-22](image)

**FIGURE 3-22**
A typical digital pulse generation circuit board from a pulse-width-modulated (PWM) induction motor drive. (Courtesy of MagneTek Drives and Systems.)

A relaxation oscillator (or pulse generator) using a PNPN diode.

**FIGURE 3-23**
A relaxation oscillator (or pulse generator) using a PNPN diode.

shown here are examples of only the most primitive types of pulse-producing circuits—more advanced ones are beyond the scope of this book.

**A Relaxation Oscillator Using a PNPN Diode**

Figure 3–23 shows a relaxation oscillator or pulse-generating circuit built with a PNPN diode. In order for this circuit to work, the following conditions must be true:

1. The power supply voltage $V_{DC}$ must exceed $V_{BO}$ for the PNPN diode.
2. $V_{DC}/R_1$ must be less than $I_H$ for the PNPN diode.
3. $R_1$ must be much larger than $R_2$.

When the switch in the circuit is first closed, capacitor $C$ will charge through resistor $R_1$ with time constant $\tau = R_1C$. As the voltage on the capacitor builds up, it will eventually exceed $V_{BO}$ and the PNPN diode will turn on. Once
the PNPN diode turns on, the capacitor will discharge through it. The discharge will be very rapid because $R_2$ is very small compared to $R_1$. Once the capacitor is discharged, the PNPN diode will turn off, since the steady-state current coming through $R_1$ is less than the current $I_H$ of the PNPN diode.

The voltage across the capacitor and the resulting output voltage and current are shown in Figure 3–24a and b, respectively.

The timing of these pulses can be changed by varying $R_1$. Suppose that resistor $R_1$ is decreased. Then the capacitor will charge more quickly, and the PNPN diode will be triggered sooner. The pulses will thus occur closer together (see Figure 3–24c).
This circuit can be used to trigger an SCR directly by removing $R_2$ and connecting the SCR gate lead in its place (see Figure 3–25a). Alternatively, the pulse circuit can be coupled to the SCR through a transformer, as shown in Figure 3–25b. If more gate current is needed to drive the SCR or TRIAC, then the pulse can be amplified by an extra transistor stage, as shown in Figure 3–25c.

The same basic circuit can also be built by using a DIAC in place of the PNPN diode (see Figure 3–26). It will function in exactly the same fashion as previously described.

In general, the quantitative analysis of pulse generation circuits is very complex and beyond the scope of this book. However, one simple example using a relaxation oscillator follows. It may be skipped with no loss of continuity, if desired.
**Example 3-2.** Figure 3-27 shows a simple relaxation oscillator using a PNPN diode. In this circuit,

\[ V_{DC} = 120 \text{ V} \quad R_1 = 100 \text{ k}\Omega \]
\[ C = 1 \mu\text{F} \quad R_2 = 1 \text{ k}\Omega \]
\[ V_{BO} = 75 \text{ V} \quad I_H = 10 \text{ mA} \]

(a) Determine the firing frequency of this circuit.

(b) Determine the firing frequency of this circuit if \( R_1 \) is increased to 150 k\(\Omega\).

**Solution**

(a) When the PNPN diode is turned off, capacitor \( C \) charges through resistor \( R_1 \) with a time constant \( \tau = R_1 C \), and when the PNPN diode turns on, capacitor \( C \) discharges through resistor \( R_2 \) with time constant \( \tau = R_2 C \). (Actually, the discharge rate is controlled by the parallel combination of \( R_1 \) and \( R_2 \), but since \( R_1 >> R_2 \), the parallel combination is essentially the same as \( R_2 \) itself.) From elementary circuit theory, the equation for the voltage on the capacitor as a function of time during the charging portion of the cycle is

\[ v_C(t) = A + B e^{-t/\tau} \]

(b) The firing frequency \( f \) of a relaxation oscillator is given by

\[ f = \frac{1}{\tau} \]

For the given circuit, \( \tau = R_1 C \), so

\[ f = \frac{1}{R_1 C} \]

For \( R_1 = 100 \text{ k}\Omega \) and \( C = 1 \mu\text{F} \),

\[ f = \frac{1}{100 \times 10^3 \times 1 \times 10^{-6}} = 10 \text{ Hz} \]

If \( R_1 \) is increased to 150 k\(\Omega\),

\[ f = \frac{1}{150 \times 10^3 \times 1 \times 10^{-6}} = \frac{1}{150} \times \frac{1}{100 \times 10^3 \times 1 \times 10^{-6}} = \frac{1}{150} \times 10 \text{ Hz} = 0.0667 \text{ Hz} \]
where \( A \) and \( B \) are constants depending on the initial conditions in the circuit. Since \( v_C(0) = 0 \) V and \( v_C(\infty) = V_{DC} \), it is possible to solve for \( A \) and \( B \):

\[
A = v_C(\infty) = V_{DC}
\]

\[
A + B = v_C(0) = 0 \Rightarrow B = -V_{DC}
\]

Therefore,

\[
v_C(t) = V_{DC} - V_{DC} e^{-\frac{t}{R_1C}} \tag{3-4}
\]

The time at which the capacitor will reach the breakover voltage is found by solving for time \( t \) in Equation (3-4):

\[
t_1 = -R_1C \ln \frac{V_{DC} - V_{BO}}{V_{DC}} \tag{3-5}
\]

In this case,

\[
t_1 = -(100 \, \text{k}\Omega)(1 \, \text{\mu}F) \ln \frac{120 \, \text{V} - 75 \, \text{V}}{120 \, \text{V}}
\]

\[
= 98 \, \text{ms}
\]

Similarly, the equation for the voltage on the capacitor as a function of time during the discharge portion of the cycle turns out to be

\[
v_C(t) = V_{BO} e^{-\frac{t}{R_2C}} \tag{3-6}
\]

so the current flow through the PNPN diode becomes

\[
i(t) = \frac{V_{BO}}{R_2} e^{-\frac{t}{R_2C}} \tag{3-7}
\]

If we ignore the continued trickle of current through \( R_1 \), the time at which \( i(t) \) reaches \( I_H \) and the PNPN diode turns off is

\[
t_2 = -R_2C \ln \frac{I_H R_2}{V_{BO}} \tag{3-8}
\]

\[
= -(1 \, \text{k}\Omega)(1 \, \text{\mu}F) \ln \frac{(10 \, \text{mA})(1 \, \text{k}\Omega)}{75 \, \text{V}} = 2 \, \text{ms}
\]

Therefore, the total period of the relaxation oscillator is

\[
T = t_1 + t_2 = 98 \, \text{ms} + 2 \, \text{ms} = 100\, \text{ms}
\]

and the frequency of the relaxation oscillator is

\[
f = \frac{1}{T} = 10 \, \text{Hz}
\]

(b) If \( R_1 \) is increased to 150 k\Omega, the capacitor charging time becomes

\[
t_1 = -R_1C \ln \frac{V_{DC} - V_{BO}}{V_{DC}}
\]

\[
= -(150 \, \text{k}\Omega)(1 \, \text{\mu}F) \ln \frac{120 \, \text{V} - 75 \, \text{V}}{120 \, \text{V}}
\]

\[
= 147 \, \text{ms}
\]
The capacitor discharging time remains unchanged at

\[ t_2 = -R_2 C \ln \frac{I_p R_2}{V_{BO}} = 2 \text{ ms} \]

Therefore, the total period of the relaxation oscillator is

\[ T = t_1 + t_2 = 147 \text{ ms} + 2 \text{ ms} = 149 \text{ ms} \]

and the frequency of the relaxation oscillator is

\[ f = \frac{1}{0.149 \text{ s}} = 6.71 \text{ Hz} \]

**Pulse Synchronization**

In ac applications, it is important that the triggering pulse be applied to the controlling SCRs at the same point in each ac cycle. The way this is normally done is to synchronize the pulse circuit to the ac power line supplying power to the SCRs. This can easily be accomplished by making the power supply to the triggering circuit the same as the power supply to the SCRs.

If the triggering circuit is supplied from a half-cycle of the ac power line, the \( RC \) circuit will always begin to charge at exactly the beginning of the cycle, so the pulse will always occur at a fixed time with respect to the beginning of the cycle.

Pulse synchronization in three-phase circuits and inverters is much more complex and is beyond the scope of this book.

**3.4 VOLTAGE VARIATION BY AC PHASE CONTROL**

The level of voltage applied to a motor is one of the most common variables in motor-control applications. The SCR and the TRIAC provide a convenient technique for controlling the average voltage applied to a load by changing the phase angle at which the source voltage is applied to it.

**AC Phase Control for a DC Load Driven from an AC Source**

Figure 3–28 illustrates the concept of phase angle power control. The figure shows a voltage-phase-control circuit with a resistive dc load supplied by an ac source. The SCR in the circuit has a breakover voltage for \( i_G = 0 \) A that is greater than the highest voltage in the circuit, while the PNPN diode has a very low breakover voltage, perhaps 10 V or so. The full-wave bridge circuit ensures that the voltage applied to the SCR and the load will always be dc.

If the switch \( S_1 \) in the picture is open, then the voltage \( V_I \) at the terminals of the rectifier will just be a full-wave rectified version of the input voltage (see Figure 3–29).

If switch \( S_1 \) is shut but switch \( S_2 \) is left open, then the SCR will always be off. This is true because the voltage out of the rectifier will never exceed \( V_{BO} \) for
the SCR. Since the SCR is always an open circuit, the current through it and the load, and hence the voltage on the load, will still be zero.

Now suppose that switch $S_2$ is closed. Then, at the beginning of the first half-cycle after the switch is closed, a voltage builds up across the $RC$ network, and the capacitor begins to charge. During the time the capacitor is charging, the SCR is off, since the voltage applied to it has not exceeded $V_{BO}$. As time passes, the capacitor charges up to the breakover voltage of the PNPN diode, and the PNPN diode conducts. The current flow from the capacitor and the PNPN diode flows through the gate of the SCR, lowering $V_{BO}$ for the SCR and turning it on. When the SCR turns on, current flows through it and the load. This current flow continues for the rest of the half-cycle, even after the capacitor has discharged, since the SCR turns off only when its current falls below the holding current (since $I_H$ is a few milliamperes, this does not occur until the extreme end of the half-cycle).

At the beginning of the next half-cycle, the SCR is again off. The $RC$ circuit again charges up over a finite period and triggers the PNPN diode. The PNPN diode once more sends a current to the gate of the SCR, turning it on. Once on, the SCR remains on for the rest of the cycle again. The voltage and current waveforms for this circuit are shown in Figure 3–30.

Now for the critical question: How can the power supplied to this load be changed? Suppose the value of $R$ is decreased. Then at the beginning of each half-
cycle, the capacitor will charge more quickly, and the SCR will fire sooner. Since the SCR will be on for longer in the half-cycle, more power will be supplied to the load (see Figure 3–31). The resistor $R$ in this circuit controls the power flow to the load in the circuit.

The power supplied to the load is a function of the time that the SCR fires; the earlier that it fires, the more power will be supplied. The firing time of the SCR is customarily expressed as a firing angle, where the firing angle is the angle of the applied sinusoidal voltage at the time of firing. The relationship between the firing angle and the supplied power will be derived in Example 3–3.
AC Phase Angle Control for an AC Load

It is possible to modify the circuit in Figure 3–28 to control an ac load simply by moving the load from the dc side of the circuit to a point before the rectifiers. The resulting circuit is shown in Figure 3–32a, and its voltage and circuit waveforms are shown in Figure 3–32b.

However, there is a much easier way to make an ac power controller. If the same basic circuit is used with a DIAC in place of the PNPN diode and a TRIAC in place of the SCR, then the diode bridge circuit can be completely taken out of the circuit. Because both the DIAC and the TRIAC are two-way devices, they op-
An ac phase angle controller using a DIAC and TRIAC is shown in Figure 3–33.

Example 3–3. Figure 3–34 shows an ac phase angle controller supplying power to a resistive load. The circuit uses a TRIAC triggered by a digital pulse circuit that can provide firing pulses at any point in each half-cycle of the applied voltage $v_s(t)$. Assume that the supply voltage is 120 V rms at 60 Hz.

(a) Determine the rms voltage applied to the load as a function of the firing angle of the pulse circuit, and plot the relationship between firing angle and the supplied voltage.

(b) What firing angle would be required to supply a voltage of 75 V rms to the load?

Solution

(a) This problem is ideally suited to solution using MATLAB because it involves a repetitive calculation of the rms voltage applied to the load at many different firing angles. We will solve the problem by calculating the waveform produced by firing the TRIAC at each angle from $1^\circ$ to $179^\circ$, and calculating the rms voltage of the resulting waveform. (Note that only the positive half cycle is considered, since the negative half cycle is symmetrical.)

The first step in the solution process is to produce a MATLAB function that mimics the load voltage for any given $\omega t$ and firing angle. Function
ac_phase_controller does this. It accepts two input arguments, a normalized time \( \omega t \) in radians and a firing angle in degrees. If the time \( \omega t \) is earlier than the firing angle, the load voltage at that time will be 0 V. If the time \( \omega t \) is after the firing angle, the load voltage will be the same as the source voltage for that time.

```matlab
function volts = ac_phase_controller(wt,deg)
% Function to simulate the output of the positive half
% cycle of an ac phase angle controller with a peak
% voltage of 120 * SQRT(2) = 170 V.
% wt = Phase in radians (=omega x time)
% deg = Firing angle in degrees

% Degrees to radians conversion factor
deg2rad = pi / 180;

% Simulate the output of the phase angle controller.
if wt > deg * deg2rad;
    volts = 170 * sin(wt);
else
    volts = 0;
end
```

The next step is to write an m-file that creates the load waveform for each possible firing angle, and calculates and plots the resulting rms voltage. The m-file shown below uses function ac_phase_controller to calculate the load voltage waveform for each firing angle, and then calculates the rms voltage of that waveform.

```matlab
% M-file: volts_vs_phase_angle.m
% M-file to calculate the rms voltage applied to a load as
% a function of the phase angle firing circuit, and to
% plot the resulting relationship.

% Loop over all firing angles (1 to 179 degrees)
deg = zeros(1,179);
rms = zeros(1,179);
for ii = 1:179

    % Save firing angle
    deg(ii) = ii;

    % First, generate the waveform to analyze.
    waveform = zeros(1,180);
    for jj = 1:180
        waveform(jj) = ac_phase_controller(jj*pi/180,ii);
    end

    % Now calculate the rms voltage of the waveform
    temp = sum(waveform.^2);
    rms(ii) = sqrt(temp/180);
end
```
% Plot rms voltage of the load as a function of firing angle
plot(deg,rms);
title('Load Voltage vs. Firing Angle');
xlabel('Firing angle (deg)');
ylabel('RMS voltage (V)');
grid on;

Two examples of the waveform generated by this function are shown in Figure 3–35.

![Graph](image)

FIGURE 3-35
Waveform produced by volts_vs_phase_angle for a firing angle of (a) 45°; (b) 90°.
When this m-file is executed, the plot shown in Figure 3–36 results. Note that the earlier the firing angle, the greater the rms voltage supplied to the load. However, the relationship between firing angle and the resulting voltage is not linear, so it is not easy to predict the required firing angle to achieve a given load voltage.

(b) The firing angle required to supply 75 V to the load can be found from Figure 3–36. It is about 99°.

The Effect of Inductive Loads on Phase Angle Control

If the load attached to a phase angle controller is inductive (as real machines are), then new complications are introduced to the operation of the controller. By the nature of inductance, the current in an inductive load cannot change instantaneously. This means that the current to the load will not rise immediately on firing the SCR (or TRIAC) and that the current will not stop flowing at exactly the end of the half-cycle. At the end of the half-cycle, the inductive voltage on the load will keep the device turned on for some time into the next half-cycle, until the current flowing through the load and the SCR finally falls below $I_H$. Figure 3–37 shows the effect of this delay in the voltage and current waveforms for the circuit in Figure 3–32.

A large inductance in the load can cause two potentially serious problems with a phase controller:

1. The inductance can cause the current buildup to be so slow when the SCR is switched on that it does not exceed the holding current before the gate current disappears. If this happens, the SCR will not remain on, because its current is less than $I_H$. 

\[ I_L(t) = 2n \left( I_H - I_D \right) - nI_D \left(1 - \frac{t}{T} \right) \]
The effect of an inductive load on the current and voltage waveforms of the circuit shown in Figure 3-32.

2. If the current continues long enough before decaying to $I_H$ after the end of a given cycle, the applied voltage could build up high enough in the next cycle to keep the current going, and the SCR will never switch off.

The normal solution to the first problem is to use a special circuit to provide a longer gating current pulse to the SCR. This longer pulse allows plenty of time
for the current through the SCR to rise above $I_H$, permitting the device to remain on for the rest of the half-cycle.

A solution to the second problem is to add a free-wheeling diode. A free-wheeling diode is a diode placed across a load and oriented so that it does not conduct during normal current flow. Such a diode is shown in Figure 3–38. At the end of a half-cycle, the current in the inductive load will attempt to keep flowing in the same direction as it was going. A voltage will be built up on the load with the polarity required to keep the current flowing. This voltage will forward-bias the free-wheeling diode, and it will supply a path for the discharge current from the load. In that manner, the SCR can turn off without requiring the current of the inductor to instantly drop to zero.

### 3.5 DC-TO-DC POWER CONTROL—CHOPPERS

Sometimes it is desirable to vary the voltage available from a dc source before applying it to a load. The circuits which vary the voltage of a dc source are called dc-to-dc converters or choppers. In a chopper circuit, the input voltage is a constant dc voltage source, and the output voltage is varied by varying the fraction of the time that the dc source is connected to its load. Figure 3–39 shows the basic principle of a chopper circuit. When the SCR is triggered, it turns on and power is supplied to the load. When it turns off, the dc source is disconnected from the load.

In the circuit shown in Figure 3–39, the load is a resistor, and the voltage on the load is either $V_{DC}$ or 0. Similarly, the current in the load is either $V_{DC}/R$ or 0. It is possible to smooth out the load voltage and current by adding a series inductor to filter out some of the ac components in the waveform. Figure 3–40 shows a chopper circuit with an inductive filter. The current through the inductor increases exponentially when the SCR is on and decreases exponentially when the SCR is off. If the inductor is large, the time constant of the current changes ($\tau = L/R$) will
be long relative to the on/off cycle of the SCR and the load voltage and current will be almost constant at some average value.

In the case of ac phase controllers, the SCRs automatically turn off at the end of each half-cycle when their currents go to zero. For dc circuits, there is no point at which the current naturally falls below $I_H$, so once an SCR is turned on, it never turns off. To turn the SCR off again at the end of a pulse, it is necessary to apply a reverse voltage to it for a short time. This reverse voltage stops the current flow and turns off the SCR. Once it is off, it will not turn on again until another pulse enters the gate of the SCR. The process of forcing an SCR to turn off at a desired time is known as forced commutation.

GTO thyristors are ideally suited for use in chopper circuits, since they are self-commutating. In contrast to SCRs, GTOs can be turned off by a negative current pulse applied to their gates. Therefore, the extra circuitry needed in an SCR
A chopper circuit with an inductive filter to smooth out the load voltage and current.

Chopper circuit to turn off the SCR can be eliminated from a GTO thyristor chopper circuit (Figure 3–41a). Power transistors are also self-commutating and are used in chopper circuits that fall within their power limits (Figure 3–41b).

Chopper circuits are used with dc power systems to vary the speed of dc motors. Their greatest advantage for dc speed control compared to conventional methods is that they are more efficient than the systems (such as the Ward-Leonard system described in Chapter 6) that they replace.

**Forced Commutation in Chopper Circuits**

When SCRs are used in choppers, a forced-commutation circuit must be included to turn off the SCRs at the desired time. Most such forced-commutation circuits
 depend for their turnoff voltage on a charged capacitor. Two basic versions of capacitor commutation are examined in this brief overview:

1. Series-capacitor commutation circuits
2. Parallel-capacitor commutation circuits

**Series-Capacitor Commutation Circuits**

Figure 3–42 shows a simple dc chopper circuit with series-capacitor commutation. It consists of an SCR, a capacitor, and a load, all in series with each other.
The capacitor and load voltages in the series chopper circuit.

The capacitor has a shunt discharging resistor across it, and the load has a freewheeling diode across it.

The SCR is initially turned on by a pulse applied to its gate. When the SCR turns on, a voltage is applied to the load and a current starts flowing through it. But this current flows through the series capacitor on the way to the load, and the capacitor gradually charges up. When the capacitor’s voltage nearly reaches $V_{DC}$, the current through the SCR drops below $I_H$ and the SCR turns off.

Once the capacitor has turned off the SCR, it gradually discharges through resistor $R$. When it is totally discharged, the SCR is ready to be fired by another pulse at its gate. The voltage and current waveforms for this circuit are shown in Figure 3-43.

Unfortunately, this type of circuit is limited in terms of duty cycle, since the SCR cannot be fired again until the capacitor has discharged. The discharge time depends on the time constant $\tau = RC$, and $C$ must be made large in order to let a lot of current flow to the load before it turns off the SCR. But $R$ must be large, since the current leaking through the resistor has to be less than the holding current of the SCR. These two facts taken together mean that the SCR cannot be refired quickly after it turns off. It has a long recovery time.

An improved series-capacitor commutation circuit with a shortened recovery time is shown in Figure 3-44. This circuit is similar to the previous one except that the resistor has been replaced by an inductor and SCR in series. When SCR is fired, current will flow to the load and the capacitor will charge up, cutting off SCR$_1$. Once it is cut off, SCR$_2$ can be fired, discharging the capacitor much more...


quickly than the resistor would. The inductor in series with SCR₂ protects SCR₂ from instantaneous current surges that exceed its ratings. Once the capacitor discharges, SCR₂ turns off and SCR₁ is ready to fire again.

**Parallel-Capacitor Commutation Circuits**

The other common way to achieve forced commutation is via the parallel-capacitor commutation scheme. A simple example of the parallel-capacitor scheme is shown in Figure 3–45. In this scheme, SCR₁ is the main SCR, supplying power to the load, and SCR₂ controls the operation of the commutating capacitor. To apply power to the load, SCR₁ is fired. When this occurs, a current flows through the SCR to the load, supplying power to it. Also, capacitor C charges up through resistor R to a voltage equal to the supply voltage $V_{DC}$.

When the time comes to turn off the power to the load, SCR₂ is fired. When SCR₂ is fired, the voltage across it drops to zero. Since the voltage across a
capacitor cannot change instantaneously, the voltage on the left side of the capacitor must instantly drop to $-V_{DC}$ volts. This turns off SCR$_1$, and the capacitor charges through the load and SCR$_2$ to a voltage of $V_{DC}$ volts positive on its left side. Once capacitor $C$ is charged, SCR$_2$ turns off, and the cycle is ready to begin again.

Again, resistor $R_1$ must be large in order for the current through it to be less than the holding current of SCR$_2$. But a large resistor $R_1$ means that the capacitor will charge only slowly after SCR$_1$ fires. This limits how soon SCR$_1$ can be turned off after it fires, setting a lower limit on the on time of the chopped waveform.

A circuit with a reduced capacitor charging time is shown in Figure 3-46. In this circuit SCR$_3$ is triggered at the same time as SCR$_1$ is, and the capacitor can
charge much more rapidly. This allows the current to be turned off much more rapidly if it is desired to do so.

In any circuit of this sort, the free-wheeling diode is extremely important. When SCR is forced off, the current through the inductive load must have another path available to it, or it could possibly damage the SCR.

3.6 INVERTERS

Perhaps the most rapidly growing area in modern power electronics is static frequency conversion, the conversion of ac power at one frequency to ac power at another frequency by means of solid-state electronics. Traditionally there have been two approaches to static ac frequency conversion: the cycloconverter and the rectifier-inverter. The cycloconverter is a device for directly converting ac power at one frequency to ac power at another frequency, while the rectifier-inverter first converts ac power to dc power and then converts the dc power to ac power again at a different frequency. This section deals with the operation of rectifier-inverter circuits, and Section 3.7 deals with the cycloconverter.

A rectifier-inverter is divided into two parts:

1. A rectifier to produce dc power
2. An inverter to produce ac power from the dc power.

Each part is treated separately.

The Rectifier

The basic rectifier circuits for converting ac power to dc power are described in Section 3.2. These circuits have one problem from a motor-control point of view—their output voltage is fixed for a given input voltage. This problem can be overcome by replacing the diodes in these circuits with SCRs.

Figure 3–47 shows a three-phase full-wave rectifier circuit with the diodes in the circuits replaced by SCRs. The average dc output voltage from this circuit depends on when the SCRs are triggered during their positive half-cycles. If they are triggered at the beginning of the half-cycle, this circuit will be the same as that of a three-phase full-wave rectifier with diodes. If the SCRs are never triggered, the output voltage will be 0 V. For any other firing angle between $0^\circ$ and $180^\circ$ on the waveform, the dc output voltage will be somewhere between the maximum value and 0 V.

When SCRs are used instead of diodes in the rectifier circuit to get control of the dc voltage output, this output voltage will have more harmonic content than a simple rectifier would, and some form of filter on its output is important. Figure 3–47 shows an inductor and capacitor filter placed at the output of the rectifier to help smooth the dc output.
ELECTRIC MACHINERY FUNDAMENTALS

FIGURE 3-47
A three-phase rectifier circuit using SCRs to provide control of the dc output voltage level.

External Commutation Inverters

Inverters are classified into two basic types by the commutation technique used: external commutation and self-commutation. *External commutation inverters* are inverters in which the energy required to turn off the SCRs is provided by an external motor or power supply. An example of an external commutation inverter is shown in Figure 3-48. The inverter is connected to a three-phase synchronous motor, which provides the countervoltage necessary to turn off one SCR when its companion is fired.

The SCRs in this circuit are triggered in the following order: SCR₁, SCR₆, SCR₂, SCR₄, SCR₃, SCR₅. When SCR₁ fires, the internal generated voltage in the synchronous motor provides the voltage necessary to turn off SCR₃. Note that if the load were not connected to the inverter, the SCRs would never be turned off and after ½ cycle a short circuit would develop through SCR₁ and SCR₄.

This inverter is also called a *load-commutated inverter*. 

FIGURE 3-48
An external commutation inverter.
Self-Commutation Inverters

If it is not possible to guarantee that a load will always provide the proper countervoltage for commutation, then a self-commutation inverter must be used. A self-commutation inverter is an inverter in which the active SCRs are turned off by energy stored in a capacitor when another SCR is switched on. It is also possible to design self-commutation inverters using GTOs or power transistors, in which case commutation capacitors are not required.

There are three major types of self-commutation inverters: current source inverters (CSIs), voltage source inverters (VSIs), and pulse-width modulation (PWM) inverters. Current source inverters and voltage source inverters are simpler than PWM inverters and have been used for a longer time. PWM inverters require more complex control circuitry and faster switching components than CSIs and VSIs. CSIs and VSIs are discussed first. Current source inverters and voltage source inverters are compared in Figure 3-49.

In the current source inverter, a rectifier is connected to an inverter through a large series inductor $L_S$. The inductance of $L_S$ is sufficiently large that the direct current is constrained to be almost constant. The SCR current output waveform will be roughly a square wave, since the current flow $I_S$ is constrained to be nearly constant. The line-to-line voltage will be approximately triangular. It is easy to limit overcurrent conditions in this design, but the output voltage can swing widely in response to changes in load.

In the voltage source inverter, a rectifier is connected to an inverter through a series inductor $L_S$ and a parallel capacitor $C$. The capacitance of $C$ is sufficiently large that the voltage is constrained to be almost constant. The SCR line-to-line voltage output waveform will be roughly a square wave, since the voltage $V_C$ is constrained to be nearly constant. The output current flow will be approximately triangular. Voltage variations are small in this circuit, but currents can vary wildly with variations in load, and overcurrent protection is difficult to implement.

The frequency of both current and voltage source inverters can be easily changed by changing the firing pulses on the gates of the SCRs, so both inverters can be used to drive ac motors at variable speeds (see Chapter 10).

A Single-Phase Current Source Inverter

A single-phase current source inverter circuit with capacitor commutation is shown in Figure 3-50. It contains two SCRs, a capacitor, and an output transformer. To understand the operation of this circuit, assume initially that both SCRs are off. If SCR$_1$ is now turned on by a gate current, voltage $V_{DC}$ will be applied to the upper half of the transformer in the circuit. This voltage induces a voltage $V_{DC}$ in the lower half of the transformer as well, causing a voltage of $2V_{DC}$ to be built up across the capacitor. The voltages and currents in the circuit at this time are shown in Figure 3-50b.

Now SCR$_2$ is turned on. When SCR$_2$ is turned on, the voltage at the cathode of the SCR will be $V_{DC}$. Since the voltage across a capacitor cannot change
<table>
<thead>
<tr>
<th>Main circuit configuration</th>
<th>Current source inverter</th>
<th>Voltage source inverter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image1" alt="Current source inverter diagram" /></td>
<td><img src="image2" alt="Voltage source inverter diagram" /></td>
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</table>

<table>
<thead>
<tr>
<th>Type of source</th>
<th>Current source $-I_S$ almost constant</th>
<th>Voltage source $-V_S$ almost constant</th>
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<table>
<thead>
<tr>
<th>Output impedance</th>
<th>High</th>
<th>Low</th>
</tr>
</thead>
</table>

| Output waveform           | ![Line voltage diagram](image3) | ![Line voltage diagram](image4) |
|                           | ![Current diagram](image5)       | ![Current diagram](image6) |

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>1. Easy to control overcurrent conditions with this design 2. Output voltage varies widely with changes in load</th>
<th>1. Difficult to limit current because of capacitor 2. Output voltage variations small because of capacitor</th>
</tr>
</thead>
</table>

**FIGURE 3-49**
Comparison of current source inverters and voltage source inverters.

Instantaneously, this forces the voltage at the top of the capacitor to instantly become $3V_{DC}$, turning off SCR$_1$. At this point, the voltage on the bottom half of the transformer is built up positive at the bottom to negative at the top of the winding, and its magnitude is $V_{DC}$. The voltage in the bottom half induces a voltage $V_{DC}$ in the upper half of the transformer, charging the capacitor $C$ up to a voltage of $2V_{DC}$, oriented positive at the bottom with respect to the top of the capacitor. The condition of the circuit at this time is shown in Figure 3–50c.

When SCR$_1$ is fired again, the capacitor voltage cuts off SCR$_2$, and this process repeats indefinitely. The resulting voltage and current waveforms are shown in Figure 3–51.
FIGURE 3-50
(a) A simple single-phase inverter circuit. (b) The voltages and currents in the circuit when SCR₁ is triggered. (c) The voltages and currents in the circuit when SCR₂ is triggered.

A Three-Phase Current Source Inverter

Figure 3–52 shows a three-phase current source inverter. In this circuit, the six SCRs fire in the order SCR₁, SCR₆, SCR₂, SCR₄, SCR₃, SCR₅. Capacitors $C₁$ through $C₆$ provide the commutation required by the SCRs.
Plots of the voltages and current in the inverter circuit: $V_1$ is the voltage at the cathode of SCR$_1$, and $V_2$ is the voltage at the cathode of SCR$_2$. Since the voltage at their anodes is $V_{DC}$, any time $V_1$ or $V_2$ exceeds $V_{DC}$, that SCR is turned off. $i_{load}$ is the current supplied to the inverter's load.
To understand the operation of this circuit, examine Figure 3–53. Assume that initially SCR₁ and SCR₅ are conducting, as shown in Figure 3–53a. Then a voltage will build up across capacitors C₁, C₃, C₄, and C₅ as shown on the diagram. Now assume that SCR₆ is gated on. When SCR₆ is turned on, the voltage at point 6 drops to zero (see Figure 3–53b). Since the voltage across capacitor C₅ cannot change instantaneously, the anode of SCR₅ is biased negative, and SCR₅ is turned off. Once SCR₆ is on, all the capacitors charge up as shown in Figure 3–53c, and the circuit is ready to turn off SCR₆ whenever SCR₄ is turned on. This same commutation process applies to the upper SCR bank as well.

The output phase and line current from this circuit are shown in Figure 3–53d.

A Three-Phase Voltage Source Inverter

Figure 3–54 shows a three-phase voltage source inverter using power transistors as the active elements. Since power transistors are self-commutating, no special commutation components are included in this circuit.
FIGURE 3-53
The operation of the three-phase CSI. (a) Initially, SCR₁ and SCR₃ are conducting. Note how the commutating capacitors have charged up. (b) The situation when SCR₆ fires. The voltage at the anode of SCR₆ falls almost instantaneously to zero. Since the voltage across capacitor C₃ cannot change instantaneously, the voltage at the anode of SCR₃ will become negative, and SCR₃ will turn off.
(c) Now SCR₁ and SCR₆ are conducting, and the commutating capacitors charge up as shown. (d) The gating pulses, SCR conducting intervals, and the output current from this inverter.
Pulse-Width Modulation Inverters

Pulse-width modulation is the process of modifying the width of the pulses in a pulse train in direct proportion to a small control signal; the greater the control voltage, the wider the resulting pulses become. By using a sinusoid of the desired frequency as the control voltage for a PWM circuit, it is possible to produce a high-power waveform whose average voltage varies sinusoidally in a manner suitable for driving ac motors.

The basic concepts of pulse-width modulation are illustrated in Figure 3–55. Figure 3–55a shows a single-phase PWM inverter circuit using IGBTs. The states of IGBT 1 through IGBT 4 in this circuit are controlled by the two comparators shown in Figure 3–55b.

A comparator is a device that compares the input voltage $v_{\text{in}}(t)$ to a reference signal and turns transistors on or off depending on the results of the test. Comparator A compares $v_{\text{in}}(t)$ to the reference voltage $v_x(t)$ and controls IGBTs $T_1$ and $T_2$ based on the results of the comparison. Comparator B compares $v_{\text{in}}(t)$ to the reference voltage $v_y(t)$ and controls IGBTs $T_3$ and $T_4$ based on the results of the comparison. If $v_{\text{in}}(t)$ is greater than $v_x(t)$ at any given time $t$, then comparator A will turn on $T_1$ and turn off $T_2$. Otherwise, it will turn off $T_1$ and turn on $T_2$. Similarly, if $v_{\text{in}}(t)$ is greater than $v_y(t)$ at any given time $t$, then comparator B will turn...
FIGURE 3-54 (concluded)

(b) The output phase and line voltages from the inverter.
The basic concepts of pulse-width modulation. (a) A single-phase PWM circuit using IGBTs.

off $T_3$ and turn on $T_4$. Otherwise, it will turn on $T_3$ and turn off $T_4$. The reference voltages $v_c(t)$ and $v_f(t)$ are shown in Figure 3-55c.

To understand the overall operation of this PWM inverter circuit, see what happens when different control voltages are applied to it. First, assume that the control voltage is 0 V. Then voltages $v_c(t)$ and $v_f(t)$ are identical, and the load voltage out of the circuit $v_{\text{load}}(t)$ is zero (see Figure 3-56).

Next, assume that a constant positive control voltage equal to one-half of the peak reference voltage is applied to the circuit. The resulting output voltage is a train of pulses with a 50 percent duty cycle, as shown in Figure 3-57.

Finally, assume that a sinusoidal control voltage is applied to the circuit as shown in Figure 3-58. The width of the resulting pulse train varies sinusoidally with the control voltage. The result is a high-power output waveform whose average voltage over any small region is directly proportional to the average voltage of the control signal in that region. The fundamental frequency of the output waveform is the same as the frequency of the input control voltage. Of course, there are harmonic components in the output voltage, but they are not usually a concern in motor-control applications. The harmonic components may cause additional heating in the motor being driven by the inverter, but the extra heating can be compensated for either by buying a specially designed motor or by derating an ordinary motor (running it at less than its full rated power).

A complete three-phase PWM inverter would consist of three of the single-phase inverters described above with control voltages consisting of sinusoids
shifted by 120° between phases. Frequency control in a PWM inverter of this sort is accomplished by changing the frequency of the input control voltage.

A PWM inverter switches states many times during a single cycle of the resulting output voltage. At the time of this writing, reference voltages with frequencies as high as 12 kHz are used in PWM inverter designs, so the components in a PWM inverter must change states up to 24,000 times per second. This rapid switching means that PWM inverters require faster components than CSIs or VSIs. PWM inverters need high-power high-frequency components such as GTO thyristors,
FIGURE 3-56
The output of the PWM circuit with an input voltage of 0 V. Note that \( v_s(t) = v_i(t) \), so \( v_{\text{load}}(t) = 0 \).
The output of the PWM circuit with an input voltage equal to one-half of the peak comparator voltage.

\[ V_{\text{load}} = v_y - v_u \]
FIGURE 3-58
The output of the PWM circuit with a sinusoidal control voltage applied to its input.
IGBTs, and/or power transistors for proper operation. (At the time of this writing, IGBTs have the advantage for high-speed, high-power switching, so they are the preferred component for building PWM inverters.) The control voltage fed to the comparator circuits is usually implemented digitally by means of a microcomputer mounted on a circuit board within the PWM motor controller. The control voltage (and therefore the output pulse width) can be controlled by the microcomputer in a manner much more sophisticated than that described here. It is possible for the microcomputer to vary the control voltage to achieve different frequencies and voltage levels in any desired manner. For example, the microcomputer could implement various acceleration and deceleration ramps, current limits, and voltage-versus-frequency curves by simply changing options in software.

A real PWM-based induction motor drive circuit is described in Section 7.10.

3.7 CYCLOCONVERTERS

The cycloconverter is a device for directly converting ac power at one frequency to ac power at another frequency. Compared to rectifier-inverter schemes, cycloconverters have many more SCRs and much more complex gating circuitry. Despite these disadvantages, cycloconverters can be less expensive than rectifier-inverters at higher power ratings.

Cycloconverters are now available in constant-frequency and variable-frequency versions. A constant-frequency cycloconverter is used to supply power at one frequency from a source at another frequency (e.g., to supply 50-Hz loads from a 60-Hz source). Variable-frequency cycloconverters are used to provide a variable output voltage and frequency from a constant-voltage and constant-frequency source. They are often used as ac induction motor drives.

Although the details of a cycloconverter can become very complex, the basic idea behind the device is simple. The input to a cycloconverter is a three-phase source which consists of three voltages equal in magnitude and phase-shifted from each other by 120°. The desired output voltage is some specified waveform, usually a sinusoid at a different frequency. The cycloconverter generates its desired output waveform by selecting the combination of the three input phases which most closely approximates the desired output voltage at each instant of time.

There are two major categories of cycloconverters, *noncirculating current cycloconverters* and *circulating current cycloconverters*. These types are distinguished by whether or not a current circulates internally within the cycloconverter; they have different characteristics. The two types of cycloconverters are described following an introduction to basic cycloconverter concepts.

**Basic Concepts**

A good way to begin the study of cycloconverters is to take a closer look at the three-phase full-wave bridge rectifier circuit described in Section 3.2. This circuit is shown in Figure 3–59 attached to a resistive load. In that figure, the diodes are divided into two halves, a positive half and a negative half. In the positive half, the
A three-phase full-wave diode bridge circuit connected to a resistive load.

diode with the highest voltage applied to it at any given time will conduct, and it will reverse-bias the other two diodes in the section. In the negative half, the diode with the lowest voltage applied to it at any given time will conduct, and it will reverse-bias the other two diodes in the section. The resulting output voltage is shown in Figure 3–59.

Now suppose that the six diodes in the bridge circuit are replaced by six SCRs as shown in Figure 3–61. Assume that initially SCR1 is conducting as shown in Figure 3–61b. This SCR will continue to conduct until the current through it falls below $I_{th}$. If no other SCR in the positive half is triggered, then SCR1 will be turned off when voltage $v_A$ goes to zero and reverses polarity at point 2. However, if SCR2 is triggered at any time after point 1, then SCR1 will be instantly reverse-biased and turned off. The process in which SCR2 forces SCR1 to turn off is called forced commutation; it can be seen that forced commutation is possible only for the phase angles between points 1 and 2. The SCRs in the negative half behave in a similar manner, as shown in Figure 3–61c. Note that if each of the SCRs is fired as soon as commutation is possible, then the output of this bridge circuit will be the same as the output of the full-wave diode bridge rectifier shown in Figure 3–59.

Now suppose that it is desired to produce a linearly decreasing output voltage with this circuit, as shown in Figure 3–62. To produce such an output, the conducting SCR in the positive half of the bridge circuit must be turned off whenever its voltage falls too far below the desired value. This is done by triggering another SCR voltage above the desired value. Similarly, the conducting SCR in the negative half of the bridge circuit must be turned off whenever its voltage rises too far above the desired value. By triggering the SCRs in the positive and negative halves at the right time, it is possible to produce an output voltage which decreases in a manner roughly corresponding to the desired waveform. It is obvious from examining Figure 3–62 that many harmonic components are present in the resulting output voltage.

**FIGURE 3–59**

$V_A(t) = V_M \sin \omega t$ V

$V_B(t) = V_M \sin (\omega t - 120^\circ)$ V

$V_C(t) = V_M \sin (\omega t - 240^\circ)$ V
Figure 3-60

(a) The signal voltage from the positive half diodes. (b) The output voltage from the negative half diodes. (c) The total voltage applied to the load.

\( V_A \)
A three-phase full-wave SCR bridge circuit connected to a resistive load. (b) The operation of the positive half of the SCRs. Assume that initially SCR\(_1\) is conducting. If SCR\(_2\) is triggered at any time after point 1, then SCR\(_1\) will be reverse-biased and shut off. (c) The operation of the negative half of the SCRs. Assume that initially SCR\(_6\) is conducting. If SCR\(_4\) is triggered at any time after point 1, then SCR\(_6\) will be reverse-biased and shut off.
FIGURE 3-62
Approximating a linearly decreasing voltage with the three-phase full-wave SCR bridge circuit.
Positive group

\[ \text{SCR}_1 \quad \text{SCR}_2 \quad \text{SCR}_3 \quad \text{SCR}_4 \]

Negative group

\[ \text{SCR}_7 \quad \text{SCR}_8 \quad \text{SCR}_9 \]

\[ v_A(t) \quad v_B(t) \quad v_C(t) \]

\[ v_L(t) \quad \text{Load} \]

\[ i(t) \]

**FIGURE 3–63**

One phase of a noncirculating current cycloconverter circuit.

If two of these SCR bridge circuits are connected in parallel with opposite polarities, the result is a noncirculating current cycloconverter.

**Noncirculating Current Cycloconverters**

One phase of a typical noncirculating current cycloconverter is shown in Figure 3–63. A full three-phase cycloconverter consists of three identical units of this type. Each unit consists of two three-phase full-wave SCR bridge circuits, one conducting current in the positive direction (the *positive group*) and one conducting current in the negative direction (the *negative group*). The SCRs in these circuits are triggered so as to approximate a sinusoidal output voltage, with the SCRs in the positive group being triggered when the current flow is in the positive direction and the SCRs in the negative group being triggered when the current flow is in the negative direction. The resulting output voltage is shown in Figure 3–64.

As can be seen from Figure 3–64, noncirculating current cycloconverters produce an output voltage with a fairly large harmonic component. These high harmonics limit the output frequency of the cycloconverter to a value less than about one-third of the input frequency.

In addition, note that current flow must switch from the positive group to the negative group or vice versa as the load current reverses direction. The cycloconverter pulse-control circuits must detect this current transition with a current polarity detector and switch from triggering one group of SCRs to triggering the other group. There is generally a brief period during the transition in which neither the positive nor the negative group is conducting. This current pause causes additional glitches in the output waveform.

The high harmonic content, low maximum frequency, and current glitches associated with noncirculating current cycloconverters combine to limit their use.
In any practical noncirculating current cycloconverter, a filter (usually a series inductor or a transformer) is placed between the output of the cycloconverter and the load, to suppress some of the output harmonics.

**Circulating Current Cycloconverters**

One phase of a typical circulating current cycloconverter is shown in Figure 3–65. It differs from the noncirculating current cycloconverter in that the positive and negative groups are connected through two large inductors, and the load is supplied from center taps on the two inductors. Unlike the noncirculating current cycloconverter, both the positive and the negative groups are conducting at the same time, and a circulating current flows around the loop formed by the two groups and the series inductors. The series inductors must be quite large in a circuit of this sort to limit the circulating current to a safe value.

The output voltage from the circulating current cycloconverter has a smaller harmonic content than the output voltage from the noncirculating current cycloconverter, and its maximum frequency can be much higher. It has a low power factor due to the large series inductors, so a capacitor is often used for power-factor compensation.

The reason that the circulating current cycloconverter has a lower harmonic content is shown in Figure 3–66. Figure 3–66a shows the output voltage of the positive group, and Figure 3–66b shows the output voltage of the negative group. The output voltage $v_{\text{load}}(t)$ across the center taps of the inductors is

$$ v_{\text{load}}(t) = \frac{v_{\text{pos}}(t) - v_{\text{neg}}(t)}{2} $$  \hspace{1cm} \text{(3–9)}
One phase of a six-pulse type of circulating current cycloconverter.

\[ v_{\text{load}}(t) = \frac{v_{\text{pos}}(t) - v_{\text{neg}}(t)}{2} \]

FIGURE 3-65
Three-phase isolation transformer
Many of the high-frequency harmonic components which appear when the positive and negative groups are examined separately are common to both groups. As such, they cancel during the subtraction and do not appear at the terminals of the cycloconverter.

Some recirculating current cycloconverters are more complex than the one shown in Figure 3-65. With more sophisticated designs, it is possible to make cycloconverters whose maximum output frequency can be even higher than their input frequency. These more complex devices are beyond the scope of this book.

**FIGURE 3-66**

Voltages in the six-pulse circulating current cycloconverter. (a) The voltage out of the positive group; (b) the voltage out of the negative group.
(c) the resulting load voltage.

### 3.8 HARMONIC PROBLEMS

Power electronic components and circuits are so flexible and useful that equipment controlled by them now makes up 50 to 60 percent of the total load on most power systems in the developed world. As a result, the behavior of these power electronic circuits strongly influences the overall operation of the power systems that they are connected to.

The principal problem associated with power electronics is the harmonic components of voltage and current induced in the power system by the switching transients in power electronic controllers. These harmonics increase the total current flows in the lines (especially in the neutral of a three-phase power system). The extra currents cause increased losses and increased heating in power system components, requiring larger components to supply the same total load. In addition, the high neutral currents can trip protective relays, shutting down portions of a power system.

As an example of this problem, consider a balanced three-phase motor with a wye connection that draws 10 A at full load. When this motor is connected to a power system, the currents flowing in each phase will be equal in magnitude and 120° out of phase with each other, and the return current in the neutral will be 0 (see Figure 3–67). Now consider the same motor supplied with the same total power through a rectifier-inverter that produces pulses of current. The currents in the power line now are shown in Figure 3–68. Note that the rms current of each line is still 10 A, but the neutral also has an rms current of 15 A! The current in the neutral consists entirely of harmonic components.
FIGURE 3-67
Current flow for a balanced three-phase, wye-connected motor:
(a) phase \( a \); (b) phase \( b \); (c) phase \( c \);
d) neutral. The rms current flow in phases \( a \), \( b \), and \( c \) is 10 A, and the
current flow in the neutral is 0.
Current flow for a balanced three-phase, wye-connected motor connected to the power line through a power electronic controller that produces current pulses: (a) phase $a$; (b) phase $b$; (c) phase $c$; (d) neutral. The rms current flow in phases $a$, $b$, and $c$ is 10 A, while the rms current flow in the neutral is 15 A.
The spectra of the currents in the three phases and in the neutral are shown in Figure 3-69. For the motor connected directly to the line, only the fundamental frequency is present in the phases, and nothing at all is present in the neutral. For the motor connected through the power controller, the current in the phases includes both the fundamental frequency and all of the odd harmonics. The current in the neutral consists principally of the third, ninth, and fifteenth harmonics.

Since power electronic circuits are such a large fraction of the total load on a modern power system, their high harmonic content causes significant problems for the power system as a whole. New standards* have been created to limit the amount of harmonics produced by power electronic circuits, and new controllers are designed to minimize the harmonics that they produce.

3.9 SUMMARY

Power electronic components and circuits have produced a major revolution in the area of motor controls during the last 35 years or so. Power electronics provide a convenient way to convert ac power to dc power, to change the average voltage level of a dc power system, to convert dc power to ac power, and to change the frequency of an ac power system.

The conversion of ac to dc power is accomplished by rectifier circuits, and the resulting dc output voltage level can be controlled by changing the firing times of the devices (SCRs, TRIACs, GTO thyristors, etc.) in the rectifier circuit.

Adjustment of the average dc voltage level on a load is accomplished by chopper circuits, which control the fraction of time for which a fixed dc voltage is applied to a load.

Static frequency conversion is accomplished by either rectifier-inverters or cycloconverters. Inverters are of two basic types: externally commutated and self-commutated. Externally commutated inverters rely on the attached load for commutation voltages; self-commutated inverters either use capacitors to produce the required commutation voltages or use self-commutating devices such as GTO thyristors. Self-commutated inverters include current source inverters, voltage source inverters, and pulse-width modulation inverters.

Cycloconverters are used to directly convert ac power at one frequency to ac power at another frequency. There are two basic types of cycloconverters: non-circulating current and circulating current. Noncirculating current cycloconverters have large harmonic components and are restricted to relatively low frequencies. In addition, they can suffer from glitches during current direction changes. Circulating current cycloconverters have lower harmonic components and are capable of operating at higher frequencies. They require large series inductors to limit the circulating current to a safe value, and so they are bulkier than noncirculating current cycloconverters of the same rating.

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*See IEC 1000-3-2. EMC: Part 3, Section 2. "Limits for harmonic current emission (equipment input current ≤ 16 A per phase)." and ANSI/IEEE Standard 519-1992, "IEEE recommended practices and requirements for harmonic control in power systems."
FIGURE 3-69
(a) The spectrum of the phase current in the balanced three-phase, wye-connected motor connected directly to the power line. Only the fundamental frequency is present. (b) The spectrum of the phase current in the balanced three-phase, wye-connected motor connected through a power electronic controller that produces current pulses. The fundamental frequency and all odd harmonics are present. (c) The neutral current for the motor connected through a electronic power controller. The third, ninth, and fifteenth harmonics are present in the current.
QUESTIONS

3-1. Explain the operation and sketch the output characteristic of a diode.
3-2. Explain the operation and sketch the output characteristic of a PNPN diode.
3-3. How does an SCR differ from a PNPN diode? When does an SCR conduct?
3-4. What is a GTO thyristor? How does it differ from an ordinary three-wire thyristor (SCR)?
3-5. What is an IGBT? What are its advantages compared to other power electronic devices?
3-6. What is a DIAC? A TRIAC?
3-7. Does a single-phase full-wave rectifier produce a better or worse dc output than a three-phase half-wave rectifier? Why?
3-8. Why are pulse-generating circuits needed in motor controllers?
3-9. What are the advantages of digital pulse-generating circuits compared to analog pulse-generating circuits?
3-10. What is the effect of changing resistor $R$ in Figure 3-32? Explain why this effect occurs.
3-11. What is forced commutation? Why is it necessary in dc-to-dc power-control circuits?
3-12. What device(s) could be used to build dc-to-dc power-control circuits without forced commutation?
3-13. What is the purpose of a free-wheeling diode in a control circuit with an inductive load?
3-14. What is the effect of an inductive load on the operation of a phase angle controller?
3-15. Can the on time of a chopper with series-capacitor commutation be made arbitrarily long? Why or why not?
3-16. Can the on time of a chopper with parallel-capacitor commutation be made arbitrarily long? Why or why not?
3-17. What is a rectifier-inverter? What is it used for?
3-18. What is a current-source inverter?
3-19. What is a voltage-source inverter? Contrast the characteristics of a VSI with those of a CSI.
3-20. What is pulse-width modulation? How do PWM inverters compare to CSI and VSI inverters?
3-21. Are power transistors more likely to be used in PWM inverters or in CSI inverters? Why?

PROBLEMS

3-1. Calculate the ripple factor of a three-phase half-wave rectifier circuit, both analytically and using MATLAB.
3-2. Calculate the ripple factor of a three-phase full-wave rectifier circuit, both analytically and using MATLAB.
3-3. Explain the operation of the circuit shown in Figure P3–1. What would happen in this circuit if switch $S$, were closed?
3-4. What would the rms voltage on the load in the circuit in Figure P3-1 be if the firing angle of the SCR were (a) 0°, (b) 30°, (c) 90°?

*3-5. For the circuit in Figure P3-1, assume that $V_{BO}$ for the DIAC is 30 V, $C_1$ is 1 μF, $R$ is adjustable in the range 1 to 20 kΩ, and switch $S_1$ is open. What is the firing angle of the circuit when $R$ is 10 kΩ? What is the rms voltage on the load under these conditions? (Caution: This problem is hard to solve analytically because the voltage charging the capacitor varies as a function of time.)

3-6. One problem with the circuit shown in Figure P3-1 is that it is very sensitive to variations in the input voltage $v_{ac}(t)$. For example, suppose the peak value of the input voltage were to decrease. Then the time that it takes capacitor $C_1$ to charge up to the breakover voltage of the DIAC will increase, and the SCR will be triggered later in each half-cycle. Therefore, the rms voltage supplied to the load will be reduced both by the lower peak voltage and by the later firing. This same effect happens in the opposite direction if $v_{ac}(t)$ increases. How could this circuit be modified to reduce its sensitivity to variations in input voltage?

3-7. Explain the operation of the circuit shown in Figure P3-2, and sketch the output voltage from the circuit.

3-8. Figure P3-3 shows a relaxation oscillator with the following parameters:

$$\begin{align*}
R_1 &= \text{variable} \\
C &= 1 \mu\text{F} \\
V_{BO} &= 30 \text{ V} \\
R_2 &= 1500 \Omega \\
V_{DC} &= 100 \text{ V} \\
I_H &= 0.5 \text{ mA}
\end{align*}$$

(a) Sketch the voltages $v_C(t)$, $v_P(t)$, and $v_0(t)$ for this circuit.
(b) If $R_1$ is currently set to 500 kΩ, calculate the period of this relaxation oscillator.

3-9. In the circuit in Figure P3-4, $T_1$ is an autotransformer with the tap exactly in the center of its winding. Explain the operation of this circuit. Assuming that the load is inductive, sketch the voltage and current applied to the load. What is the purpose of SCR2? What is the purpose of $D_2$? (This chopper circuit arrangement is known as a Jones circuit.)

*The asterisk in front of a problem number indicates that it is a more difficult problem.
FIGURE P3-2
The inverter circuit of Problem 3-7.

FIGURE P3-3
The relaxation oscillator circuit of Problem 3-8.

FIGURE P3-4
The chopper circuit of Problem 3-9.
3-10. A series-capacitor forced commutation chopper circuit supplying a purely resistive load is shown in Figure P3-5.

\[ V_{DC} = 120 \text{ V} \quad R_1 = 20 \text{ k}\Omega \]
\[ I_H = 8 \text{ mA} \quad R_{load} = 250 \text{ } \Omega \]
\[ V_{BO} = 200 \text{ V} \quad C = 150 \text{ } \mu\text{F} \]

(a) When SCR\textsubscript{1} is turned on, how long will it remain on? What causes it to turn off?
(b) When SCR\textsubscript{1} turns off, how long will it be until the SCR can be turned on again?
   (Assume that 3 time constants must pass before the capacitor is discharged.)
(c) What problem or problems do these calculations reveal about this simple series-capacitor forced-commutation chopper circuit?
(d) How can the problem(s) described in part c be eliminated?

![Diagram of a series-capacitor forced-commutation chopper circuit]

**FIGURE P3-5**

3-11. A parallel-capacitor forced-commutation chopper circuit supplying a purely resistive load is shown in Figure P3-6.

\[ V_{DC} = 120 \text{ V} \quad R_1 = 20 \text{ k}\Omega \]
\[ I_H = 5 \text{ mA} \quad R_{load} = 250 \text{ } \Omega \]
\[ V_{BO} = 250 \text{ V} \quad C = 15 \text{ } \mu\text{F} \]

(a) When SCR\textsubscript{1} is turned on, how long will it remain on? What causes it to turn off?
(b) What is the earliest time that SCR\textsubscript{1} can be turned off after it is turned on?
   (Assume that 3 time constants must pass before the capacitor is charged.)
(c) When SCR\textsubscript{1} turns off, how long will it be until the SCR can be turned on again?
(d) What problem or problems do these calculations reveal about this simple parallel-capacitor forced-commutation chopper circuit?
(e) How can the problem(s) described in part d be eliminated?

3-12. Figure P3–7 shows a single-phase rectifier-inverter circuit. Explain how this circuit functions. What are the purposes of \( C_1 \) and \( C_2 \)? What controls the output frequency of the inverter?
*3–13. A simple full-wave ac phase angle voltage controller is shown in Figure P3–8. The component values in this circuit are

\[
R = 20 \text{ to } 300 \text{ k}\Omega, \text{ currently set to } 80 \text{ k}\Omega \\
C = 0.15 \mu F \\
V_{BO} = 40 \text{ V (for PNPN diode } D_1) \\
V_{BO} = 250 \text{ V (for SCR}_1) \\
v_S(t) = V_M \sin \omega t \text{ V where } V_M = 169.7 \text{ V and } \omega = 377 \text{ rad/s} \\
(a) \text{ At what phase angle do the PNPN diode and the SCR turn on?} \\
(b) \text{ What is the rms voltage supplied to the load under these circumstances?}

*3–14. Figure P3–9 shows a three-phase full-wave rectifier circuit supplying power to a dc load. The circuit uses SCRs instead of diodes as the rectifying elements.
The full-wave phase angle voltage controller of Problem 3–13.

The three-phase full-wave rectifier circuit of Problem 3–14.

(a) What will the rms load voltage and ripple be if each SCR is triggered as soon as it becomes forward-biased? At what phase angle should the SCRs be triggered in order to operate this way? Sketch or plot the output voltage for this case.

(b) What will the rms load voltage and ripple be if each SCR is triggered at a phase angle of 90° (that is, halfway through the half-cycle in which it is forward-biased)? Sketch or plot the output voltage for this case.

*3–15. Write a MATLAB program that imitates the operation of the pulse-width modulation circuit shown in Figure 3–55, and answer the following questions.

(a) Assume that the comparison voltages \( v_{j}(t) \) and \( v_{i}(t) \) have peak amplitudes of 10 V and a frequency of 500 Hz. Plot the output voltage when the input voltage is \( v_{in}(t) = 10 \sin 2\pi ft \) V, and \( f = 60 \text{ Hz} \).

(b) What does the spectrum of the output voltage look like? What could be done to reduce the harmonic content of the output voltage?

(c) Now assume that the frequency of the comparison voltages is increased to 1000 Hz. Plot the output voltage when the input voltage is \( v_{in}(t) = 10 \sin 2\pi ft \) V and \( f = 60 \text{ Hz} \).

(d) What does the spectrum of the output voltage in c look like?

(e) What is the advantage of using a higher comparison frequency and more rapid switching in a PWM modulator?
4.6 WINDING INSULATION IN AN AC MACHINE

One of the most critical parts of an ac machine design is the insulation of its windings. If the insulation of a motor or generator breaks down, the machine shorts out. The repair of a machine with shorted insulation is quite expensive, if it is even possible. To prevent the winding insulation from breaking down as a result of overheating, it is necessary to limit the temperature of the windings. This can be partially done by providing a cooling air circulation over them, but ultimately the maximum winding temperature limits the maximum power that can be supplied continuously by the machine.
Insulation rarely fails from immediate breakdown at some critical temperature. Instead, the increase in temperature produces a gradual degradation of the insulation, making it subject to failure from another cause such as shock, vibration, or electrical stress. There was an old rule of thumb that said that the life expectancy of a motor with a given type of insulation is halved for each 10 percent rise in temperature above the rated temperature of the winding. This rule still applies to some extent today.

To standardize the temperature limits of machine insulation, the National Electrical Manufacturers Association (NEMA) in the United States has defined a series of *insulation system classes*. Each insulation system class specifies the maximum temperature rise permissible for that class of insulation. There are three common NEMA insulation classes for integral-horsepower ac motors: B, F, and H. Each class represents a higher permissible winding temperature than the one before it. For example, the armature winding temperature rise above ambient temperature in one type of continuously operating ac induction motor must be limited to 80°C for class B, 105°C for class F, and 125°C for class H insulation.

The effect of operating temperature on insulation life for a typical machine can be quite dramatic. A typical curve is shown in Figure 4–20. This curve shows the mean life of a machine in thousands of hours versus the temperature of the windings, for several different insulation classes.

The specific temperature specifications for each type of ac motor and generator are set out in great detail in NEMA Standard MG1-1993, *Motors and Generators*. Similar standards have been defined by the International Electrotechnical Commission (IEC) and by various national standards organizations in other countries.
FIGURE 4-20
Plot of mean insulation life versus winding temperature for various insulation classes. (Courtesy of Marathon Electric Company.)
7.5 INDUCTION MOTOR TORQUE–SPEED CHARACTERISTICS

How does the torque of an induction motor change as the load changes? How much torque can an induction motor supply at starting conditions? How much does the speed of an induction motor drop as its shaft load increases? To find out the answers to these and similar questions, it is necessary to clearly understand the relationships among the motor's torque, speed, and power.

In the following material, the torque–speed relationship will be examined first from the physical viewpoint of the motor's magnetic field behavior. Then, a general equation for torque as a function of slip will be derived from the induction motor equivalent circuit (Figure 7–12).

Induced Torque from a Physical Standpoint

Figure 7–15a shows a cage rotor induction motor that is initially operating at no load and therefore very nearly at synchronous speed. The net magnetic field $B_{\text{net}}$ in this machine is produced by the magnetization current $I_M$ flowing in the motor's equivalent circuit (see Figure 7–12). The magnitude of the magnetization current and hence of $B_{\text{net}}$ is directly proportional to the voltage $E_1$. If $E_1$ is constant, then the
net magnetic field in the motor is constant. In an actual machine, $E_I$ varies as the load changes, because the stator impedances $R_I$ and $X_I$ cause varying voltage drops with varying load. However, these drops in the stator windings are relatively small, so $E_I$ (and hence $I_M$ and $B_{net}$) is approximately constant with changes in load.

Figure 7-15a shows the induction motor at no load. At no load, the rotor slip is very small, and so the relative motion between the rotor and the magnetic fields is very small and the rotor frequency is also very small. Since the relative motion is small, the voltage $E_R$ induced in the bars of the rotor is very small, and the resulting current flow $I_R$ is small. Also, because the rotor frequency is so very small, the reactance of the rotor is nearly zero, and the maximum rotor current $I_R$ is almost in phase with the rotor voltage $E_R$. The rotor current thus produces a small magnetic field $B_R$ at an angle just slightly greater than $90^\circ$ behind the net magnetic field $B_{net}$. Notice that the stator current must be quite large even at no load, since it must supply most of $B_{net}$ (This is why induction motors have large no-load currents compared to other types of machines.)

The induced torque, which keeps the rotor turning, is given by the equation

$$\tau_{ind} = kB_R \times B_{net}$$  \hspace{1cm} (4-60)

Its magnitude is given by

$$\tau_{ind} = kB_R B_{net} \sin \delta$$  \hspace{1cm} (4-61)

Since the rotor magnetic field is very small, the induced torque is also quite small—just large enough to overcome the motor's rotational losses.

Now suppose the induction motor is loaded down (Figure 7-15b). As the motor's load increases, its slip increases, and the rotor speed falls. Since the rotor speed is slower, there is now more relative motion between the rotor and the sta-
tor magnetic fields in the machine. Greater relative motion produces a stronger rotor voltage $E_R$ which in turn produces a larger rotor current $I_R$. With a larger rotor current, the rotor magnetic field $B_R$ also increases. However, the angle of the rotor current and $B_R$ changes as well. Since the rotor slip is larger, the rotor frequency rises ($f_s = sf_e$), and the rotor’s reactance increases ($\omega L_{R}$). Therefore, the rotor current now lags further behind the rotor voltage, and the rotor magnetic field shifts with the current. Figure 7-15b shows the induction motor operating at a fairly high load. Notice that the rotor current has increased and that the angle $\delta$ has increased. The increase in $B_R$ tends to increase the torque, while the increase in angle $\delta$ tends to decrease the torque ($\tau_{\text{ind}}$ is proportional to $\sin \delta$, and $\delta > 90^\circ$). Since the first effect is larger than the second one, the overall induced torque increases to supply the motor’s increased load.

When does an induction motor reach pullout torque? This happens when the point is reached where, as the load on the shaft is increased, the $\sin \delta$ term decreases more than the $B_R$ term increases. At that point, a further increase in load decreases $\tau_{\text{ind}}$, and the motor stops.

It is possible to use a knowledge of the machine’s magnetic fields to approximately derive the output torque-versus-speed characteristic of an induction motor. Remember that the magnitude of the induced torque in the machine is given by

$$\tau_{\text{ind}} = kB_R B_{\text{net}} \sin \delta$$

Each term in this expression can be considered separately to derive the overall machine behavior. The individual terms are

1. $B_R$. The rotor magnetic field is directly proportional to the current flowing in the rotor, as long as the rotor is unsaturated. The current flow in the rotor increases with increasing slip (decreasing speed) according to Equation (7-13). This current flow was plotted in Figure 7-11 and is shown again in Figure 7-16a.

2. $B_{\text{net}}$. The net magnetic field in the motor is proportional to $E_1$ and therefore is approximately constant ($E_1$ actually decreases with increasing current flow, but this effect is small compared to the other two, and it will be ignored in this graphical development). The curve for $B_{\text{net}}$ versus speed is shown in Figure 7-16b.

3. $\sin \delta$. The angle $\delta$ between the net and rotor magnetic fields can be expressed in a very useful way. Look at Figure 7-15b. In this figure, it is clear that the angle $\delta$ is just equal to the power-factor angle of the rotor plus $90^\circ$:

$$\delta = \theta_R + 90^\circ$$

Therefore, $\sin \delta = \sin (\theta_R + 90^\circ) = \cos \theta_R$. This term is the power factor of the rotor. The rotor power-factor angle can be calculated from the equation

$$\theta_R = \tan^{-1} \frac{X_R}{R_R} = \tan^{-1} \frac{X_{R0}}{R_R}$$

(7-39)
The resulting rotor power factor is given by

\[ PF_R = \cos \theta_R \]

\[ PF_R = \cos \left( \tan^{-1} \frac{sX_R}{R_R} \right) \]  \hspace{1cm} (7–40)

A plot of rotor power factor versus speed is shown in Figure 7–16c.

Since the induced torque is proportional to the product of these three terms, the torque–speed characteristic of an induction motor can be constructed from the
graphical multiplication of the previous three plots (Figure 7–16a to c). The torque–speed characteristic of an induction motor derived in this fashion is shown in Figure 7–16d.

This characteristic curve can be divided roughly into three regions. The first region is the low-slip region of the curve. In the low-slip region, the motor slip increases approximately linearly with increased load, and the rotor mechanical speed decreases approximately linearly with load. In this region of operation, the rotor reactance is negligible, so the rotor power factor is approximately unity, while the rotor current increases linearly with slip. The entire normal steady-state operating range of an induction motor is included in this linear low-slip region. Thus in normal operation, an induction motor has a linear speed droop.

The second region on the induction motor’s curve can be called the moderate-slip region. In the moderate-slip region, the rotor frequency is higher than before, and the rotor reactance is on the same order of magnitude as the rotor resistance. In this region, the rotor current no longer increases as rapidly as before, and the power factor starts to drop. The peak torque (the pullout torque) of the motor occurs at the point where, for an incremental increase in load, the increase in the rotor current is exactly balanced by the decrease in the rotor power factor.

The third region on the induction motor’s curve is called the high-slip region. In the high-slip region, the induced torque actually decreases with increased load, since the increase in rotor current is completely overshadowed by the decrease in rotor power factor.

For a typical induction motor, the pullout torque on the curve will be 200 to 250 percent of the rated full-load torque of the machine, and the starting torque (the torque at zero speed) will be 150 percent or so of the full-load torque. Unlike a synchronous motor, the induction motor can start with a full load attached to its shaft.

The Derivation of the Induction Motor Induced-Torque Equation

It is possible to use the equivalent circuit of an induction motor and the power-flow diagram for the motor to derive a general expression for induced torque as a function of speed. The induced torque in an induction motor is given by Equation (7–35) or (7–36):

\[ \tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} \]  
(7–35)

\[ \tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} \]  
(7–36)

The latter equation is especially useful, since the synchronous speed is a constant for a given frequency and number of poles. Since \( \omega_{\text{sync}} \) is constant, a knowledge of the air-gap power gives the induced torque of the motor.

The air-gap power is the power crossing the gap from the stator circuit to the rotor circuit. It is equal to the power absorbed in the resistance \( R_2/s \). How can this power be found?
Refer to the equivalent circuit given in Figure 7–17. In this figure, the air-gap power supplied to one phase of the motor can be seen to be

\[ P_{AG,1\phi} = I_2^2 \frac{R_2}{s} \]

Therefore, the total air-gap power is

\[ P_{AG} = 3I_2^2 \frac{R_2}{s} \]

If \( I_2 \) can be determined, then the air-gap power and the induced torque will be known.

Although there are several ways to solve the circuit in Figure 7–17 for the current \( I_2 \), perhaps the easiest one is to determine the Thevenin equivalent of the portion of the circuit to the left of the \( X' \)s in the figure. Thevenin's theorem states that any linear circuit that can be separated by two terminals from the rest of the system can be replaced by a single voltage source in series with an equivalent impedance. If this were done to the induction motor equivalent circuit, the resulting circuit would be a simple series combination of elements as shown in Figure 7–18c.

To calculate the Thevenin equivalent of the input side of the induction motor equivalent circuit, first open-circuit the terminals at the \( X' \)s and find the resulting open-circuit voltage present there. Then, to find the Thevenin impedance, kill (short-circuit) the phase voltage and find the \( Z_{eq} \) seen “looking” into the terminals.

Figure 7–18a shows the open terminals used to find the Thevenin voltage. By the voltage divider rule,

\[ V_{TH} = V_\phi \frac{Z_M}{Z_M + Z_l} \]

\[ = V_\phi \frac{jX_M}{R_1 + jX_1 + jX_M} \]

The magnitude of the Thevenin voltage \( V_{TH} \) is

\[ V_{TH} = V_\phi \sqrt{R_1^2 + (X_1 + X_M)^2} \]

(7–41a)
Since the magnetization reactance $X_M \gg X_1$ and $X_M \gg R_1$, the magnitude of the Thevenin voltage is approximately

$$V_{TH} \approx V_\phi \frac{X_M}{X_1 + X_M} \quad (7-41b)$$

to quite good accuracy.

Figure 7–18b shows the input circuit with the input voltage source killed. The two impedances are in parallel, and the Thevenin impedance is given by

$$Z_{TH} = \frac{Z_1 Z_M}{Z_1 + Z_M} \quad (7-42)$$

This impedance reduces to
Because \( X_M \gg X_1 \) and \( X_M + X_1 \gg R_1 \), the Thevenin resistance and reactance are approximately given by

\[
R_{TH} \approx R_1 \left( \frac{X_M}{X_1 + X_M} \right)^2 \quad (7-44)
\]

\[
X_{TH} \approx X_1 \quad (7-45)
\]

The resulting equivalent circuit is shown in Figure 7–18c. From this circuit, the current \( I_2 \) is given by

\[
I_2 = \frac{V_{TH}}{Z_{TH} + Z_2} = \frac{V_{TH}}{R_{TH} + R_2/s + jX_{TH} + jX_2} \quad (7-46)
\]

The magnitude of this current is

\[
I_2 = \frac{V_{TH}}{\sqrt{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}} \quad (7-48)
\]

The air-gap power is therefore given by

\[
P_{AG} = 3I_2^2 \frac{R_2}{s} = \frac{3V_{TH}^2 R_2}{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2} \quad (7-49)
\]

and the rotor-induced torque is given by

\[
\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{3V_{TH}^2 R_2}{\omega_{sync}[(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2]} \quad (7-50)
\]

A plot of induction motor torque as a function of speed (and slip) is shown in Figure 7–19, and a plot showing speeds both above and below the normal motor range is shown in Figure 7–20.

**Comments on the Induction Motor Torque-Speed Curve**

The induction motor torque–speed characteristic curve plotted in Figures 7–19 and 7–20 provides several important pieces of information about the operation of induction motors. This information is summarized as follows:
1. The induced torque of the motor is zero at synchronous speed. This fact has been discussed previously.

2. The torque–speed curve is nearly linear between no load and full load. In this range, the rotor resistance is much larger than the rotor reactance, so the rotor current, the rotor magnetic field, and the induced torque increase linearly with increasing slip.

3. There is a maximum possible torque that cannot be exceeded. This torque, called the pullout torque or breakdown torque, is 2 to 3 times the rated full-load torque of the motor. The next section of this chapter contains a method for calculating pullout torque.

4. The starting torque on the motor is slightly larger than its full-load torque, so this motor will start carrying any load that it can supply at full power.

5. Notice that the torque on the motor for a given slip varies as the square of the applied voltage. This fact is useful in one form of induction motor speed control that will be described later.

6. If the rotor of the induction motor is driven faster than synchronous speed, then the direction of the induced torque in the machine reverses and the machine becomes a generator, converting mechanical power to electric power. The use of induction machines as generators will be described later.
7. If the motor is turning backward relative to the direction of the magnetic fields, the induced torque in the machine will stop the machine very rapidly and will try to rotate it in the other direction. Since reversing the direction of magnetic field rotation is simply a matter of switching any two stator phases, this fact can be used as a way to very rapidly stop an induction motor. The act of switching two phases in order to stop the motor very rapidly is called plugging.

The power converted to mechanical form in an induction motor is equal to

\[ P_{\text{conv}} = \tau_{\text{ind}} \omega_m \]

and is shown plotted in Figure 7–21. Notice that the peak power supplied by the induction motor occurs at a different speed than the maximum torque; and, of course, no power is converted to mechanical form when the rotor is at zero speed.

**Maximum (Pullout) Torque in an Induction Motor**

Since the induced torque is equal to \( P_{AG}/\omega_{sync} \), the maximum possible torque occurs when the air-gap power is maximum. Since the air-gap power is equal to the power consumed in the resistor \( R_2/s \), the maximum induced torque will occur when the power consumed by that resistor is maximum.
When is the power supplied to $R_2/s$ at its maximum? Refer to the simplified equivalent circuit in Figure 7–18c. In a situation where the angle of the load impedance is fixed, the maximum power transfer theorem states that maximum power transfer to the load resistor $R_2/s$ will occur when the magnitude of that impedance is equal to the magnitude of the source impedance. The equivalent source impedance in the circuit is

$$Z_{\text{source}} = R_{\text{TH}} + jX_{\text{TH}} + jX_2$$

so the maximum power transfer occurs when

$$\frac{R_2}{s} = \sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}$$

(7–52)

Solving Equation (7–52) for slip, we see that the slip at pullout torque is given by

$$s_{\text{max}} = \frac{R_2}{\sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}}$$

(7–53)

Notice that the referred rotor resistance $R_2$ appears only in the numerator, so the slip of the rotor at maximum torque is directly proportional to the rotor resistance.

The value of the maximum torque can be found by inserting the expression for the slip at maximum torque into the torque equation [Equation (7–50)]. The resulting equation for the maximum or pullout torque is

\[
\tau_{\text{max}} = \frac{3V_{\text{TH}}^2}{2\omega_{\text{sync}}[R_{\text{TH}}^2 + \sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}]} \quad (7–54)
\]

This torque is proportional to the square of the supply voltage and is also inversely related to the size of the stator impedances and the rotor reactance. The smaller a machine’s reactances, the larger the maximum torque it is capable of achieving. Note that slip at which the maximum torque occurs is directly proportional to rotor resistance [Equation (7–53)], but the value of the maximum torque is independent of the value of rotor resistance [Equation (7–54)].

The torque–speed characteristic for a wound-rotor induction motor is shown in Figure 7–22. Recall that it is possible to insert resistance into the rotor circuit of a wound rotor because the rotor circuit is brought out to the stator through slip rings. Notice on the figure that as the rotor resistance is increased, the pullout speed of the motor decreases, but the maximum torque remains constant.
It is possible to take advantage of this characteristic of wound-rotor induction motors to start very heavy loads. If a resistance is inserted into the rotor circuit, the maximum torque can be adjusted to occur at starting conditions. Therefore, the maximum possible torque would be available to start heavy loads. On the other hand, once the load is turning, the extra resistance can be removed from the circuit, and the maximum torque will move up to near-synchronous speed for regular operation.

Example 7–4. A two-pole, 50-Hz induction motor supplies 15 kW to a load at a speed of 2950 r/min.

(a) What is the motor's slip?
(b) What is the induced torque in the motor in N • m under these conditions?
(c) What will the operating speed of the motor be if its torque is doubled?
(d) How much power will be supplied by the motor when the torque is doubled?

Solution
(a) The synchronous speed of this motor is

\[ n_{\text{sync}} = \frac{120f}{P} = \frac{120(50 \text{ Hz})}{2 \text{ poles}} = 3000 \text{ r/min} \]

Therefore, the motor's slip is

\[ s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} \times 100\% \]

\[ = \frac{3000 \text{ r/min} - 2950 \text{ r/min}}{3000 \text{ r/min}} \times 100\% \]

\[ = 0.0167 \text{ or } 1.67\% \]

(b) The induced torque in the motor must be assumed equal to the load torque, and \( P_{\text{conv}} \) must be assumed equal to \( P_{\text{load}} \), since no value was given for mechanical losses. The torque is thus

\[ \tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{15 \text{ kW}}{(2950 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min/60 s})} = 48.6 \text{ N • m} \]

(c) In the low-slip region, the torque–speed curve is linear, and the induced torque is directly proportional to slip. Therefore, if the torque doubles, then the new slip will be 3.33 percent. The operating speed of the motor is thus

\[ n_m = (1 - s)n_{\text{sync}} = (1 - 0.0333)(3000 \text{ r/min}) = 2900 \text{ r/min} \]

(d) The power supplied by the motor is given by

\[ P_{\text{conv}} = \tau_{\text{ind}}\omega_m \]

\[ = (97.2 \text{ N • m})(2900 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min/60 s}) \]

\[ = 29.5 \text{ kW} \]
Example 7–5. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit:

\[
\begin{align*}
R_1 &= 0.641 \, \Omega \\
R_2 &= 0.332 \, \Omega \\
X_1 &= 1.106 \, \Omega \\
X_2 &= 0.464 \, \Omega \\
X_M &= 26.3 \, \Omega
\end{align*}
\]

(a) What is the maximum torque of this motor? At what speed and slip does it occur?

(b) What is the starting torque of this motor?

(c) When the rotor resistance is doubled, what is the speed at which the maximum torque now occurs? What is the new starting torque of the motor?

(d) Calculate and plot the torque–speed characteristics of this motor both with the original rotor resistance and with the rotor resistance doubled.

Solution

The Thevenin voltage of this motor is

\[
V_{TH} = V_f \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}
\]

\[
= \frac{(266 \, \text{V})(26.3 \, \Omega)}{\sqrt{(0.641 \, \Omega)^2 + (1.106 \, \Omega + 26.3 \, \Omega)^2}} = 255.2 \, \text{V}
\]

The Thevenin resistance is

\[
R_{TH} \approx R_1 \left(\frac{X_M}{X_1 + X_M}\right)^2
\]

\[
\approx (0.641 \, \Omega) \left(\frac{26.3 \, \Omega}{1.106 \, \Omega + 26.3 \, \Omega}\right)^2 = 0.590 \, \Omega
\]

The Thevenin reactance is

\[
X_{TH} \approx X_1 = 1.106 \, \Omega
\]

(a) The slip at which maximum torque occurs is given by Equation (7–53):

\[
s_{\text{max}} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}
\]

\[
= \frac{0.332 \, \Omega}{\sqrt{(0.590 \, \Omega)^2 + (1.106 \, \Omega + 0.464 \, \Omega)^2}} = 0.198
\]

This corresponds to a mechanical speed of

\[
\omega_m = (1 - s) \omega_{\text{sync}} = (1 - 0.198)(1800 \, \text{r/min}) = 1444 \, \text{r/min}
\]

The torque at this speed is

\[
\tau_{\text{max}} = \frac{3V_{TH}^2}{2\omega_{\text{sync}} [R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}]}
\]

\[
= \frac{3(255.2 \, \text{V})^2}{2(188.5 \, \text{rad/s})(0.590 \, \Omega + \sqrt{(0.590 \, \Omega)^2 + (1.106 \, \Omega + 0.464 \, \Omega)^2})}
\]

\[
= 229 \, \text{N} \cdot \text{m}
\]
(b) The starting torque of this motor is found by setting $s = 1$ in Equation (7–50):

$$
\tau_{\text{start}} = \frac{3V_{TH}^2 R_2}{\omega_{\text{sync}}[(R_{TH} + R_2)^2 + (X_{TH} + X_2)^2]}
$$

$$
= \frac{3(255.2 \text{ V})^2(0.332 \Omega)}{(188.5 \text{ rad/s})[(0.590 \Omega + 0.332 \Omega)^2 + (1.106 \Omega + 0.464 \Omega)^2]}
$$

$$
= 104 \text{ N} \cdot \text{m}
$$

(c) If the rotor resistance is doubled, then the slip at maximum torque doubles, too. Therefore,

$$
s_{\text{max}} = 0.396
$$

and the speed at maximum torque is

$$
n_m = (1 - s)n_{\text{sync}} = (1 - 0.396)(1800 \text{ r/min}) = 1087 \text{ r/min}
$$

The maximum torque is still

$$
\tau_{\text{max}} = 229 \text{ N} \cdot \text{m}
$$

The starting torque is now

$$
\tau_{\text{start}} = \frac{3(255.2 \text{ V})^2(0.664 \Omega)}{(188.5 \text{ rad/s})[(0.590 \Omega + 0.664 \Omega)^2 + (1.106 \Omega + 0.464 \Omega)^2]}
$$

$$
= 170 \text{ N} \cdot \text{m}
$$

(d) We will create a MATLAB M-file to calculate and plot the torque–speed characteristic of the motor both with the original rotor resistance and with the doubled rotor resistance. The M-file will calculate the Thevenin impedance using the exact equations for $V_{TH}$ and $Z_{TH}$ [Equations (7–41a) and (7–43)] instead of the approximate equations, because the computer can easily perform the exact calculations. It will then calculate the induced torque using Equation (7–50) and plot the results. The resulting M-file is shown below:

```matlab
% M-file: torque_speed_curve.m
% M-file create a plot of the torque-speed curve of the
% induction motor of Example 7-5.

% First, initialize the values needed in this program.
rl = 0.641; % Stator resistance
x1 = 1.106; % Stator reactance
r2 = 0.332; % Rotor resistance
x2 = 0.464; % Rotor reactance
xm = 26.3; % Magnetization branch reactance
v_phase = 460 / sqrt(3); % Phase voltage
n_sync = 1800; % Synchronous speed (r/min)
w_sync = 188.5; % Synchronous speed (rad/s)

% Calculate the Thevenin voltage and impedance from Equations % 7-41a and 7-43.
v_th = v_phase * (xm / sqrt(rl^2 + (x1 + xm)^2));
z_th = ((j*xm) * (rl + j*x1)) / (rl + j*(x1 + xm));
r_th = real(z_th);
x_th = imag(z_th);
```
Now calculate the torque-speed characteristic for many slips between 0 and 1. Note that the first slip value \( s \) is set to 0.001 instead of exactly 0 to avoid divide-by-zero problems.

\[
s = (0:1:50) / 50; \quad \% \text{ Slip}
\]

\[
s(1) = 0.001; \quad \% \text{ Mechanical speed}
\]

\[
\text{s(1)} = 0.001;
\]

\[
\text{nm} = (1 - \text{s}) * \text{n_sync}; \quad \% \text{ Mechanical speed}
\]

\[
\text{Calculate torque for original rotor resistance}
\]

\[
\text{for ii = 1:51}
\]

\[
\text{t_ind1}(\text{ii}) = \left( 3 * \text{v_th}^2 * \text{r2} / \text{s(ii)} \right) / ... \\
\quad \left( \text{w_sync} * \left( (\text{r_th} + \text{r2}/\text{s(ii)})^2 + (\text{x_th} + \text{x2})^2 \right) \right); \\
\]

\[
\text{end}
\]

\[
\text{Calculate torque for doubled rotor resistance}
\]

\[
\text{for ii = 1:51}
\]

\[
\text{t_ind2}(\text{ii}) = \left( 3 * \text{v_th}^2 * (2*\text{r2}) / \text{s(ii)} \right) / ... \\
\quad \left( \text{w Sync} * \left( (\text{r_th} + (2*\text{r2})/\text{s(ii)})^2 + (\text{x_th} + \text{x2})^2 \right) \right); \\
\]

\[
\text{end}
\]

\[
\text{Plot the torque-speed curve}
\]

\[
\text{plot(nm,}\text{t_ind1},'\text{Color}', 'k','\text{LineWidth}',2.0);} \\
\text{hold on;}
\]

\[
\text{plot(nm,}\text{t_ind2},'\text{Color}', 'k','\text{LineWidth}',2.0,'\text{LineStyle}', '-. ');}
\]

\[
\text{xlabel('\text{\textit{m}}','\text{Fontweight}', 'Bold');}
\]

\[
\text{ylabel('\text{\tau_{ind}}','\text{Fontweight}', 'Bold');}
\]

\[
\text{title ('\text{Induction motor torque-speed characteristic}',...}
\quad '\text{Fontweight}', 'Bold');
\]

\[
\text{legend ('Original R_{(2)}','Doubled R_{(2)}');}
\]

\[
\text{grid on;}
\]

\[
\text{hold off;}
\]

The resulting torque-speed characteristics are shown in Figure 7-23. Note that the peak torque and starting torque values on the curves match the calculations of parts (a) through (c). Also, note that the starting torque of the motor rose as \( R_2 \) increased.
7.8 STARTING INDUCTION MOTORS

Induction motors do not present the types of starting problems that synchronous motors do. In many cases, induction motors can be started by simply connecting them to the power line. However, there are sometimes good reasons for not doing this. For example, the starting current required may cause such a dip in the power system voltage that across-the-line starting is not acceptable.

For wound-rotor induction motors, starting can be achieved at relatively low currents by inserting extra resistance in the rotor circuit during starting. This extra resistance not only increases the starting torque but also reduces the starting current.

For cage induction motors, the starting current can vary widely depending primarily on the motor's rated power and on the effective rotor resistance at starting conditions. To estimate the rotor current at starting conditions, all cage motors now have a starting code letter (not to be confused with their design class letter) on their nameplates. The code letter sets limits on the amount of current the motor can draw at starting conditions.

These limits are expressed in terms of the starting apparent power of the motor as a function of its horsepower rating. Figure 7–34 is a table containing the starting kilovoltamperes per horsepower for each code letter.

To determine the starting current for an induction motor, read the rated voltage, horsepower, and code letter from its nameplate. Then the starting apparent power for the motor will be

\[ S_{\text{start}} = \text{(rated horsepower)}(\text{code letter factor}) \]  \hspace{1cm} (7-55)

and the starting current can be found from the equation

\[ I_L = \frac{S_{\text{start}}}{\sqrt{3}V_T} \]  \hspace{1cm} (7-56)

Example 7-7. What is the starting current of a 15-hp, 208-V, code-letter-F, three-phase induction motor?

Solution

According to Figure 7–34, the maximum kilovoltamperes per horsepower is 5.6. Therefore, the maximum starting kilovoltamperes of this motor is

\[ S_{\text{start}} = (15 \text{ hp})(5.6) = 84 \text{ kVA} \]
The starting current is thus

\[ I_L = \frac{S_{\text{start}}}{\sqrt{3}V_T} \]  

\[ = \frac{84 \text{kVA}}{\sqrt{3}(208 \text{ V})} = 233 \text{ A} \] (7-56)

If necessary, the starting current of an induction motor may be reduced by a starting circuit. However, if this is done, it will also reduce the starting torque of the motor.

One way to reduce the starting current is to insert extra inductors or resistors into the power line during starting. While formerly common, this approach is rare today. An alternative approach is to reduce the motor's terminal voltage during starting by using autotransformers to step it down. Figure 7–35 shows a typical reduced-voltage starting circuit using autotransformers. During starting, contacts 1 and 3 are shut, supplying a lower voltage to the motor. Once the motor is nearly up to speed, those contacts are opened and contacts 2 are shut. These contacts put full line voltage across the motor.

It is important to realize that while the starting current is reduced in direct proportion to the decrease in terminal voltage, the starting torque decreases as the square of the applied voltage. Therefore, only a certain amount of current reduction can be done if the motor is to start with a shaft load attached.
Line terminals

Motor terminals

Starting sequence:
(a) Close 1 and 3
(b) Open 1 and 3
(c) Close 2

FIGURE 7-35
An autotransformer starter for an induction motor.

FIGURE 7-36
A typical across-the-line starter for an induction motor.

Induction Motor Starting Circuits
A typical full-voltage or across-the-line magnetic induction motor starter circuit is shown in Figure 7-36, and the meanings of the symbols used in the figure are explained in Figure 7-37. This operation of this circuit is very simple. When the start button is pressed, the relay (or contactor) coil M is energized, causing the normally open contacts M₁, M₂, and M₃ to shut. When these contacts shut, power is applied to the induction motor, and the motor starts. Contact M₄ also shuts,
which shorts out the starting switch, allowing the operator to release it without removing power from the M relay. When the stop button is pressed, the M relay is deenergized, and the M contacts open, stopping the motor.

A magnetic motor starter circuit of this sort has several built-in protective features:

1. Short-circuit protection
2. Overload protection
3. Undervoltage protection

*Short-circuit protection* for the motor is provided by fuses $F_1$, $F_2$, and $F_3$. If a sudden short circuit develops within the motor and causes a current flow many times larger than the rated current, these fuses will blow, disconnecting the motor from the power supply and preventing it from burning up. However, these fuses must not burn up during normal motor starting, so they are designed to require currents many times greater than the full-load current before they open the circuit. This means that short circuits through a high resistance and/or excessive motor loads will not be cleared by the fuses.

*Overload protection* for the motor is provided by the devices labeled $OL$ in the figure. These overload protection devices consist of two parts, an overload
heater element and overload contacts. Under normal conditions, the overload contacts are shut. However, when the temperature of the heater elements rises far enough, the OL contacts open, deenergizing the M relay, which in turn opens the normally open M contacts and removes power from the motor.

When an induction motor is overloaded, it is eventually damaged by the excessive heating caused by its high currents. However, this damage takes time, and an induction motor will not normally be hurt by brief periods of high currents (such as starting currents). Only if the high current is sustained will damage occur. The overload heater elements also depend on heat for their operation, so they will not be affected by brief periods of high current during starting, and yet they will operate during long periods of high current, removing power from the motor before it can be damaged.

Undervoltage protection is provided by the controller as well. Notice from the figure that the control power for the M relay comes from directly across the lines to the motor. If the voltage applied to the motor falls too much, the voltage applied to the M relay will also fall and the relay will deenergize. The M contacts then open, removing power from the motor terminals.

An induction motor starting circuit with resistors to reduce the starting current flow is shown in Figure 7-38. This circuit is similar to the previous one, except that there are additional components present to control removal of the starting resistor. Relays 1TD, 2TD, and 3TD in Figure 7-38 are so-called time-delay relays, meaning that when they are energized there is a set time delay before their contacts shut.

When the start button is pushed in this circuit, the M relay energizes and power is applied to the motor as before. Since the 1TD, 2TD, and 3TD contacts are all open, the full starting resistor is in series with the motor, reducing the starting current.

When the M contacts close, notice that the 1TD relay is energized. However, there is a finite delay before the 1TD contacts close. During that time, the motor partially speeds up, and the starting current drops off some. After that time, the 1TD contacts close, cutting out part of the starting resistance and simultaneously energizing the 2TD relay. After another delay, the 2TD contacts shut, cutting out the second part of the resistor and energizing the 3TD relay. Finally, the 3TD contacts close, and the entire starting resistor is out of the circuit.

By a judicious choice of resistor values and time delays, this starting circuit can be used to prevent the motor starting current from becoming dangerously large, while still allowing enough current flow to ensure prompt acceleration to normal operating speeds.

7.9 SPEED CONTROL OF INDUCTION MOTORS

Until the advent of modern solid-state drives, induction motors in general were not good machines for applications requiring considerable speed control. The normal operating range of a typical induction motor (design classes A, B, and C)

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is confined to less than 5 percent slip, and the speed variation over that range is more or less directly proportional to the load on the shaft of the motor. Even if the slip could be made larger, the efficiency of the motor would become very poor, since the rotor copper losses are directly proportional to the slip on the motor (remember that $P_{RCL} = sP_{AG}$).

There are really only two techniques by which the speed of an induction motor can be controlled. One is to vary the synchronous speed, which is the speed of the stator and rotor magnetic fields, since the rotor speed always remains near $n_{syn}$.

The other technique is to vary the slip of the motor for a given load. Each of these approaches will be taken up in more detail.

The synchronous speed of an induction motor is given by
so the only ways in which the synchronous speed of the machine can be varied are (1) by changing the electrical frequency and (2) by changing the number of poles on the machine. Slip control may be accomplished by varying either the rotor resistance or the terminal voltage of the motor.

**Induction Motor Speed Control by Pole Changing**

There are two major approaches to changing the number of poles in an induction motor:

1. The method of consequent poles
2. Multiple stator windings

The *method of consequent poles* is quite an old method for speed control, having been originally developed in 1897. It relies on the fact that the number of poles in the stator windings of an induction motor can easily be changed by a factor of 2:1 with only simple changes in coil connections. Figure 7–39 shows a
simple two-pole induction motor stator suitable for pole changing. Notice that the individual coils are of very short pitch (60 to 90°). Figure 7–40 shows phase $a$ of these windings separately for more clarity of detail.

Figure 7–40a shows the current flow in phase $a$ of the stator windings at an instant of time during normal operation. Note that the magnetic field leaves the stator in the upper phase group (a north pole) and enters the stator in the lower phase group (a south pole). This winding is thus producing two stator magnetic poles.

![Connections at far end of stator](image)

**FIGURE 7–40**
A close-up view of one phase of a pole-changing winding. (a) In the two-pole configuration, one coil is a north pole and the other one is a south pole. (b) When the connection on one of the two coils is reversed, they are both north poles, and the magnetic flux returns to the stator at points halfway between the two coils. The south poles are called *consequent poles*, and the winding is now a four-pole winding.
Now suppose that the direction of current flow in the lower phase group on the stator is reversed (Figure 7-40b). Then the magnetic field will leave the stator in both the upper phase group and the lower phase group—each one will be a north magnetic pole. The magnetic flux in this machine must return to the stator between the two phase groups, producing a pair of consequent south magnetic poles. Notice that now the stator has four magnetic poles—twice as many as before.

The rotor in such a motor is of the cage design, since a cage rotor always has as many poles induced in it as there are in the stator and can thus adapt when the number of stator poles changes.

When the motor is reconnected from two-pole to four-pole operation, the resulting maximum torque of the induction motor can be the same as before (constant-torque connection), half of its previous value (square-law-torque connection, used for fans, etc.), or twice its previous value (constant-output-power connection), depending on how the stator windings are rearranged. Figure 7-41 shows the possible stator connections and their effect on the torque-speed curve.

The major disadvantage of the consequent-pole method of changing speed is that the speeds must be in a ratio of 2:1. The traditional approach to overcoming this limitation was to employ multiple stator windings with different numbers of poles and to energize only one set at a time. For example, a motor might be wound with a four-pole and a six-pole set of stator windings, and its synchronous speed on a 60-Hz system could be switched from 1800 to 1200 r/min simply by supplying power to the other set of windings. Unfortunately, multiple stator windings increase the expense of the motor and are therefore used only when absolutely necessary.

By combining the method of consequent poles with multiple stator windings, it is possible to build a four-speed induction motor. For example, with separate four- and six-pole windings, it is possible to produce a 60-Hz motor capable of running at 600, 900, 1200, and 1800 r/min.

**Speed Control by Changing the Line Frequency**

If the electrical frequency applied to the stator of an induction motor is changed, the rate of rotation of its magnetic fields \( n_{\text{sync}} \) will change in direct proportion to the change in electrical frequency, and the no-load point on the torque–speed characteristic curve will change with it (see Figure 7–42). The synchronous speed of the motor at rated conditions is known as the base speed. By using variable frequency control, it is possible to adjust the speed of the motor either above or below base speed. A properly designed variable-frequency induction motor drive can be very flexible. It can control the speed of an induction motor over a range from as little as 5 percent of base speed up to about twice base speed. However, it is important to maintain certain voltage and torque limits on the motor as the frequency is varied, to ensure safe operation.

When running at speeds below the base speed of the motor, it is necessary to reduce the terminal voltage applied to the stator for proper operation. The terminal voltage applied to the stator should be decreased linearly with decreasing
Possible connections of the stator coils in a pole-changing motor, together with the resulting torque-speed characteristics: (a) *Constant-torque connection*—the torque capabilities of the motor remain approximately constant in both high-speed and low-speed connections. (b) *Constant-horsepower connection*—the power capabilities of the motor remain approximately constant in both high-speed and low-speed connections. (c) *Fan torque connection*—the torque capabilities of the motor change with speed in the same manner as fan-type loads.
Variable-frequency speed control in an induction motor: (a) The family of torque-speed characteristic curves for speeds below base speed, assuming that the line voltage is derated linearly with frequency. (b) The family of torque-speed characteristic curves for speeds above base speed, assuming that the line voltage is held constant.
FIGURE 7-42 (concluded)
(c) The torque-speed characteristic curves for all frequencies.

To understand the necessity for derating, recall that an induction motor is basically a rotating transformer. As with any transformer, the flux in the core of an induction motor can be found from Faraday's law:

\[ v(t) = -N \frac{d\phi}{dt} \]  \hspace{1cm} (1-36)

If a voltage \( v(t) = V_M \sin \omega t \) is applied to the core, the resulting flux \( \phi \) is

\[ \phi(t) = \frac{1}{N_p} \int v(t) \, dt \]

\[ = \frac{1}{N_p} \int V_M \sin \omega t \, dt \]

\[ \phi(t) = -\frac{V_M}{\omega N_p} \cos \omega t \] \hspace{1cm} (7-57)

Note that the electrical frequency appears in the denominator of this expression. Therefore, if the electrical frequency applied to the stator decreases by 10 percent while the magnitude of the voltage applied to the stator remains constant, the flux in the core of the motor will increase by about 10 percent and the magnetization current of the motor will increase. In the unsaturated region of the motor's
magnetization curve, the increase in magnetization current will also be about 10 percent. However, in the saturated region of the motor's magnetization curve, a 10 percent increase in flux requires a much larger increase in magnetization current. Induction motors are normally designed to operate near the saturation point on their magnetization curves, so the increase in flux due to a decrease in frequency will cause excessive magnetization currents to flow in the motor. (This same problem was observed in transformers; see Section 2.12.)

To avoid excessive magnetization currents, it is customary to decrease the applied stator voltage in direct proportion to the decrease in frequency whenever the frequency falls below the rated frequency of the motor. Since the applied voltage $v$ appears in the numerator of Equation (7-57) and the frequency $\omega$ appears in the denominator of Equation (7-57), the two effects counteract each other, and the magnetization current is unaffected.

When the voltage applied to an induction motor is varied linearly with frequency below the base speed, the flux in the motor will remain approximately constant. Therefore, the maximum torque which the motor can supply remains fairly high. However, the maximum power rating of the motor must be decreased linearly with decreases in frequency to protect the stator circuit from overheating. The power supplied to a three-phase induction motor is given by

$$ P = \sqrt{3} V_L I_L \cos \theta $$

If the voltage $V_L$ is decreased, then the maximum power $P$ must also be decreased, or else the current flowing in the motor will become excessive, and the motor will overheat.

Figure 7-42a shows a family of induction motor torque–speed characteristic curves for speeds below base speed, assuming that the magnitude of the stator voltage varies linearly with frequency.

When the electrical frequency applied to the motor exceeds the rated frequency of the motor, the stator voltage is held constant at the rated value. Although saturation considerations would permit the voltage to be raised above the rated value under these circumstances, it is limited to the rated voltage to protect the winding insulation of the motor. The higher the electrical frequency above base speed, the larger the denominator of Equation (7-57) becomes. Since the numerator term is held constant above rated frequency, the resulting flux in the machine decreases and the maximum torque decreases with it. Figure 7-42b shows a family of induction motor torque–speed characteristic curves for speeds above base speed, assuming that the stator voltage is held constant.

If the stator voltage is varied linearly with frequency below base speed and is held constant at rated value above base speed, then the resulting family of torque–speed characteristics is as shown in Figure 7-42c. The rated speed for the motor shown in Figure 7-42 is 1800 r/min.

In the past, the principal disadvantage of electrical frequency control as a method of speed changing was that a dedicated generator or mechanical frequency changer was required to make it operate. This problem has disappeared with the development of modern solid-state variable-frequency motor drives.
fact, changing the line frequency with solid-state motor drives has become the method of choice for induction motor speed control. Note that this method can be used with any induction motor, unlike the pole-changing technique, which requires a motor with special stator windings.

A typical solid-state variable-frequency induction motor drive will be described in Section 7.10.

**Speed Control by Changing the Line Voltage**

The torque developed by an induction motor is proportional to the square of the applied voltage. If a load has a torque–speed characteristic such as the one shown in Figure 7–43, then the speed of the motor may be controlled over a limited range by varying the line voltage. This method of speed control is sometimes used on small motors driving fans.

**Speed Control by Changing the Rotor Resistance**

In wound-rotor induction motors, it is possible to change the shape of the torque–speed curve by inserting extra resistances into the rotor circuit of the machine. The resulting torque–speed characteristic curves are shown in Figure 7–44.
If the torque–speed curve of the load is as shown in the figure, then changing the rotor resistance will change the operating speed of the motor. However, inserting extra resistances into the rotor circuit of an induction motor seriously reduces the efficiency of the machine. Such a method of speed control is normally used only for short periods because of this efficiency problem.

7.10 SOLID-STATE INDUCTION MOTOR DRIVES

As mentioned in the previous section, the method of choice today for induction motor speed control is the solid-state variable-frequency induction motor drive. A typical drive of this sort is shown in Figure 7–45. The drive is very flexible: its input power can be either single-phase or three-phase, either 50 or 60 Hz, and anywhere from 208 to 230 V. The output from this drive is a three-phase set of voltages whose frequency can be varied from 0 up to 120 Hz and whose voltage can be varied from 0 V up to the rated voltage of the motor.

The output voltage and frequency control is achieved by using the pulse-width modulation (PWM) techniques described in Chapter 3. Both output frequency and output voltage can be controlled independently by pulse-width modulation. Figure 7–46 illustrates the manner in which the PWM drive can control the output frequency while maintaining a constant rms voltage level, while Figure 7–47 illustrates
A typical solid-state variable-frequency induction motor drive. (Courtesy of MagneTek, Inc.)

FIGURE 7-46
Variable-frequency control with a PWM waveform: (a) 60-Hz, 120-V PWM waveform; (b) 30-Hz, 120-V PWM waveform.
the manner in which the PWM drive can control the rms voltage level while maintaining a constant frequency.

As we described in Section 7.9, it is often desirable to vary the output frequency and output rms voltage together in a linear fashion. Figure 7–48 shows typical output voltage waveforms from one phase of the drive for the situation in which frequency and voltage are varied simultaneously in a linear fashion.* Figure 7–48a shows the output voltage adjusted for a frequency of 60 Hz and an rms voltage of 120 V. Figure 7–48b shows the output adjusted for a frequency of 30 Hz and an rms voltage of 60 V, and Figure 7–48c shows the output adjusted for a frequency of 20 Hz and an rms voltage of 40 V. Notice that the peak voltage out of the drive remains the same in all three cases; the rms voltage level is controlled by the fraction of time the voltage is switched on, and the frequency is controlled by the rate at which the polarity of the pulses switches from positive to negative and back again.

The typical induction motor drive shown in Figure 7–45 has many built-in features which contribute to its adjustability and ease of use. Here is a summary of some of these features.

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*aThe output waveforms in Figure 7–47 are actually simplified waveforms. The real induction motor drive has a much higher carrier frequency than that shown in the figure.
FIGURE 7-48
Simultaneous voltage and frequency control with a PWM waveform: (a) 60-Hz, 120-V PWM waveform; (b) 30-Hz, 60-V PWM waveform; (c) 20-Hz, 40-V PWM waveform.

**Frequency (Speed) Adjustment**

The output frequency of the drive can be controlled manually from a control mounted on the drive cabinet, or it can be controlled remotely by an external voltage or current signal. The ability to adjust the frequency of the drive in response to some external signal is very important, since it permits an external computer or process controller to control the speed of the motor in accordance with the overall needs of the plant in which it is installed.
A Choice of Voltage and Frequency Patterns

The types of mechanical loads which might be attached to an induction motor vary greatly. Some loads such as fans require very little torque when starting (or running at low speeds) and have torques which increase as the square of the speed. Other loads might be harder to start, requiring more than the rated full-load torque of the motor just to get the load moving. This drive provides a variety of voltage-versus-frequency patterns which can be selected to match the torque from the induction motor to the torque required by its load. Three of these patterns are shown in Figures 7-49 through 7-51.

Figure 7-49a shows the standard or general-purpose voltage-versus-frequency pattern, described in the previous section. This pattern changes the output voltage linearly with changes in output frequency for speeds below base speed and holds the output voltage constant for speeds above base speed. (The small constant-voltage region at very low frequencies is necessary to ensure that there will be some starting torque at the very lowest speeds.) Figure 7-49b shows the resulting induction motor torque-speed characteristics for several operating frequencies below base speed.

Figure 7-50a shows the voltage-versus-frequency pattern used for loads with high starting torques. This pattern also changes the output voltage linearly with changes in output frequency for speeds below base speed, but it has a shallower slope at frequencies below 30 Hz. For any given frequency below 30 Hz, the output voltage will be higher than it was with the previous pattern. This higher voltage will produce a higher torque, but at the cost of increased magnetic saturation and higher magnetization currents. The increased saturation and higher currents are often acceptable for the short periods required to start heavy loads. Figure 7-50b shows the induction motor torque-speed characteristics for several operating frequencies below base speed. Notice the increased torque available at low frequencies compared to Figure 7-49b.

Figure 7-51a shows the voltage-versus-frequency pattern used for loads with low starting torques (called soft-start loads). This pattern changes the output voltage parabolically with changes in output frequency for speeds below base speed. For any given frequency below 60 Hz, the output voltage will be lower than it was with the standard pattern. This lower voltage will produce a lower torque, providing a slow, smooth start for low-torque loads. Figure 7-51b shows the induction motor torque-speed characteristics for several operating frequencies below base speed. Notice the decreased torque available at low frequencies compared to Figure 7-49.

Independently Adjustable Acceleration and Deceleration Ramps

When the desired operating speed of the motor is changed, the drive controlling it will change frequency to bring the motor to the new operating speed. If the speed change is sudden (e.g., an instantaneous jump from 900 to 1200 r/min), the drive
FIGURE 7–49
(a) Possible voltage-versus-frequency patterns for the solid-state variable-frequency induction motor drive: general-purpose pattern. This pattern consists of a linear voltage–frequency curve below rated frequency and a constant voltage above rated frequency. (b) The resulting torque–speed characteristic curves for speeds below rated frequency (speeds above rated frequency look like Figure 7–41b).
FIGURE 7-50
(a) Possible voltage-versus-frequency patterns for the solid-state variable-frequency induction motor drive: high-starting-torque pattern. This is a modified voltage-frequency pattern suitable for loads requiring high starting torques. It is the same as the linear voltage-frequency pattern except at low speeds. The voltage is disproportionately high at very low speeds, which produces extra torque at the cost of a higher magnetization current. (b) The resulting torque-speed characteristic curves for speeds below rated frequency (speeds above rated frequency look like Figure 7-41b).
FIGURE 7–51
(a) Possible voltage-versus-frequency patterns for the solid-state variable-frequency induction motor drive: *fan torque pattern*. This is a voltage–frequency pattern suitable for use with motors driving fans and centrifugal pumps, which have a very low starting torque. (b) The resulting torque–speed characteristic curves for speeds below rated frequency (speeds above rated frequency look like Figure 7–41b).
does not try to make the motor instantaneously jump from the old desired speed to the new desired speed. Instead, the rate of motor acceleration or deceleration is limited to a safe level by special circuits built into the electronics of the drive. These rates can be adjusted independently for accelerations and decelerations.

**Motor Protection**

The induction motor drive has built into it a variety of features designed to protect the motor attached to the drive. The drive can detect excessive steady-state currents (an overload condition), excessive instantaneous currents, overvoltage conditions, or undervoltage conditions. In any of the above cases, it will shut down the motor.

Induction motor drives like the one described above are now so flexible and reliable that induction motors with these drives are displacing dc motors in many applications which require a wide range of speed variation.
7.13 INDUCTION MOTOR RATINGS

A nameplate for a typical high-efficiency integral-horsepower induction motor is shown in Figure 7–61. The most important ratings present on the nameplate are

1. Output power
2. Voltage
3. Current
4. Power factor
5. Speed
6. Nominal efficiency
7. NEMA design class
8. Starting code

A nameplate for a typical standard-efficiency induction motor would be similar, except that it might not show a nominal efficiency.

The voltage limit on the motor is based on the maximum acceptable magnetization current flow, since the higher the voltage gets, the more saturated the motor's iron becomes and the higher its magnetization current becomes. Just as in the case of transformers and synchronous machines, a 60-Hz induction motor may be used on a 50-Hz power system, but only if the voltage rating is decreased by an amount proportional to the decrease in frequency. This derating is necessary because the flux in the core of the motor is proportional to the integral of the applied
voltage. To keep the maximum flux in the core constant while the period of integration is increasing, the average voltage level must decrease.

The current limit on an induction motor is based on the maximum acceptable heating in the motor’s windings, and the power limit is set by the combination of the voltage and current ratings with the machine’s power factor and efficiency.

NEMA design classes, starting code letters, and nominal efficiencies were discussed in previous sections of this chapter.
9.8 DC MOTOR STARTERS

In order for a dc motor to function properly on the job, it must have some special control and protection equipment associated with it. The purposes of this equipment are

1. To protect the motor against damage due to short circuits in the equipment
2. To protect the motor against damage from long-term overloads
3. To protect the motor against damage from excessive starting currents
4. To provide a convenient manner in which to control the operating speed of the motor

The first three functions will be discussed in this section, and the fourth function will be considered in Section 9.9.

DC Motor Problems on Starting

In order for a dc motor to function properly, it must be protected from physical damage during the starting period. At starting conditions, the motor is not turning, and so \( E_A = 0 \) V. Since the internal resistance of a normal dc motor is very low
A shunt motor with a starting resistor in series with its armature. Contacts 1A, 2A, and 3A short-circuit portions of the starting resistor when they close.

Compared to its size (3 to 6 percent per unit for medium-size motors), a very high current flows.

Consider, for example, the 50-hp, 250-V motor in Example 9–1. This motor has an armature resistance $R_A$ of 0.06 Ω, and a full-load current less than 200 A, but the current on starting is

$$I_A = \frac{V_T - E_A}{R_A}$$

$$= \frac{250 \text{ V} - 0 \text{ V}}{0.06 \text{ Ω}} = 4167 \text{ A}$$

This current is over 20 times the motor's rated full-load current. It is possible for a motor to be severely damaged by such currents, even if they last for only a moment.

A solution to the problem of excess current during starting is to insert a starting resistor in series with the armature to limit the current flow until $E_A$ can build up to do the limiting. This resistor must not be in the circuit permanently, because it would result in excessive losses and would cause the motor's torque–speed characteristic to drop off excessively with an increase in load.

Therefore, a resistor must be inserted into the armature circuit to limit current flow at starting, and it must be removed again as the speed of the motor builds up. In modern practice, a starting resistor is made up of a series of pieces, each of which is removed from the motor circuit in succession as the motor speeds up, in order to limit the current in the motor to a safe value while never reducing it to too low a value for rapid acceleration.

Figure 9–28 shows a shunt motor with an extra starting resistor that can be cut out of the circuit in segments by the closing of the 1A, 2A, and 3A contacts. Two actions are necessary in order to make a working motor starter. The first is to pick the size and number of resistor segments necessary in order to limit the starting current to its desired bounds. The second is to design a control circuit that
shuts the resistor bypass contacts at the proper time to remove those parts of the resistor from the circuit.

Some older dc motor starters used a continuous starting resistor which was gradually cut out of the circuit by a person moving its handle (Figure 9–29). This type of starter had problems, as it largely depended on the person starting the motor not to move its handle too quickly or too slowly. If the resistance were cut out too quickly (before the motor could speed up enough), the resulting current flow would be too large. On the other hand, if the resistance were cut out too slowly, the starting resistor could burn up. Since they depended on a person for their correct operation, these motor starters were subject to the problem of human error. They have almost entirely been displaced in new installations by automatic starter circuits.

Example 9–7 illustrates the selection of the size and number of resistor segments needed by an automatic starter circuit. The question of the timing required to cut the resistor segments out of the armature circuit will be examined later.

Example 9–7. Figure 9–28 shows a 100-hp, 250-V, 350-A shunt dc motor with an armature resistance of 0.05 Ω. It is desired to design a starter circuit for this motor which will limit the maximum starting current to twice its rated value and which will switch out sections of resistance as the armature current falls to its rated value.

(a) How many stages of starting resistance will be required to limit the current to the range specified?

(b) What must the value of each segment of the resistor be? At what voltage should each stage of the starting resistance be cut out?

Solution

(a) The starting resistor must be selected so that the current flow equals twice the rated current of the motor when it is first connected to the line. As the motor starts to speed up, an internal generated voltage $E_A$ will be produced in the
motor. Since this voltage opposes the terminal voltage of the motor, the increasing internal generated voltage decreases the current flow in the motor. When the current flowing in the motor falls to rated current, a section of the starting resistor must be taken out to increase the starting current back up to 200 percent of rated current. As the motor continues to speed up, \( E_A \) continues to rise and the armature current continues to fall. When the current flowing in the motor falls to rated current again, another section of the starting resistor must be taken out. This process repeats until the starting resistance to be removed at a given stage is less than the resistance of the motor's armature circuit. At that point, the motor's armature resistance will limit the current to a safe value all by itself.

How many steps are required to accomplish the current limiting? To find out, define \( R_{\text{tot}} \) as the original resistance in the starting circuit. So \( R_{\text{tot}} \) is the sum of the resistance of each stage of the starting resistor together with the resistance of the armature circuit of the motor:

\[
R_{\text{tot}} = R_1 + R_2 + \cdots + R_A
\]  
(9–29)

Now define \( R_{\text{tot},i} \) as the total resistance left in the starting circuit after stages 1 to \( i \) have been shorted out. The resistance left in the circuit after removing stages 1 through \( i \) is

\[
R_{\text{tot},i} = R_{i+1} + \cdots + R_A
\]  
(9–30)

Note also that the initial starting resistance must be

\[
R_{\text{tot}} = \frac{V_T}{I_{\text{max}}}
\]

In the first stage of the starter circuit, resistance \( R_1 \) must be switched out of the circuit when the current \( I_A \) falls to

\[
I_A = \frac{V_T - E_A}{R_1} = I_{\text{min}}
\]

After switching that part of the resistance out, the armature current must jump to

\[
I_A = \frac{V_T - E_A}{R_{\text{tot},1}} = I_{\text{max}}
\]

Since \( E_A (= K_\phi \omega) \) is directly proportional to the speed of the motor, which cannot change instantaneously, the quantity \( V_T - E_A \) must be constant at the instant the resistance is switched out. Therefore,

\[
I_{\text{min}} R_{\text{tot}} = V_T - E_A = I_{\text{max}} R_{\text{tot},1}
\]
or the resistance left in the circuit after the first stage is switched out is

\[
R_{\text{tot},1} = \frac{I_{\text{min}}}{I_{\text{max}}} R_{\text{tot}}
\]  
(9–31)

By direct extension, the resistance left in the circuit after the \( n \)th stage is switched out is

\[
R_{\text{tot},n} = \left( \frac{I_{\text{min}}}{I_{\text{max}}} \right)^n R_{\text{tot}}
\]  
(9–32)
The starting process is completed when $R'_{\text{tot},n}$ for stage $n$ is less than or equal to the internal armature resistance $R_A$ of the motor. At that point, $R_A$ can limit the current to the desired value all by itself. At the boundary where $R_A = R_{\text{tot},n}$,

$$R_A = R_{\text{tot},n} = \left(\frac{I_{\text{min}}}{I_{\text{max}}}\right)^n R_{\text{tot}}$$  \hspace{1cm} (9-33)

$$\frac{R_A}{R_{\text{tot}}} = \left(\frac{I_{\text{min}}}{I_{\text{max}}}\right)^n$$  \hspace{1cm} (9-34)

Solving for $n$ yields

$$n = \frac{\log \left(\frac{R_A}{R_{\text{tot}}}\right)}{\log \left(\frac{I_{\text{min}}}{I_{\text{max}}}\right)}$$  \hspace{1cm} (9-35)

where $n$ must be rounded up to the next integer value, since it is not possible to have a fractional number of starting stages. If $n$ has a fractional part, then when the final stage of starting resistance is removed, the armature current of the motor will jump up to a value smaller than $I_{\text{max}}$.

In this particular problem, the ratio $I_{\text{min}}/I_{\text{max}} = 0.5$, and $R_{\text{tot}}$ is

$$R_{\text{tot}} = \frac{V_T}{I_{\text{max}}} = \frac{250 \text{ V}}{700 \text{ A}} = 0.357 \Omega$$

so

$$n = \frac{\log \left(\frac{R_A}{R_{\text{tot}}}\right)}{\log \left(\frac{I_{\text{min}}}{I_{\text{max}}}\right)} = \frac{\log (0.05 \Omega/0.357 \Omega)}{\log (350 \text{ A}/700 \text{ A})} = 2.84$$

The number of stages required will be three.

(b) The armature circuit will contain the armature resistor $R_A$ and three starting resistors $R_1$, $R_2$, and $R_3$. This arrangement is shown in Figure 9–28.

At first, $E_A = 0$ V and $I_A = 700$ A, so

$$I_A = \frac{V_T}{R_A + R_1 + R_2 + R_3} = 700 \text{ A}$$

Therefore, the total resistance must be

$$R_A + R_1 + R_2 + R_3 = \frac{250 \text{ V}}{700 \text{ A}} = 0.357 \Omega$$  \hspace{1cm} (9-36)

This total resistance will be placed in the circuit until the current falls to 350 A. This occurs when

$$E_A = V_T - I_A R_{\text{tot}} = 250 \text{ V} - (350 \text{ A})(0.357 \Omega) = 125 \text{ V}$$

When $E_A = 125$ V, $I_A$ has fallen to 350 A and it is time to cut out the first starting resistor $R_1$. When it is cut out, the current should jump back to 700 A. Therefore,

$$R_A + R_2 + R_3 = \frac{V_T - E_A}{I_{\text{max}}} = \frac{250 \text{ V} - 125 \text{ V}}{700 \text{ A}} = 0.1786 \Omega$$  \hspace{1cm} (9-37)

This total resistance will be in the circuit until $I_A$ again falls to 350 A. This occurs when $E_A$ reaches

$$E_A = V_T - I_A R_{\text{tot}} = 250 \text{ V} - (350 \text{ A})(0.1786 \Omega) = 187.5 \text{ V}$$
When $E_A = 187.5 \text{ V}$, $I_A$ has fallen to 350 A and it is time to cut out the second starting resistor $R_2$. When it is cut out, the current should jump back to 700 A. Therefore,

$$R_A + R_3 = \frac{V_T - E_A}{I_{\text{max}}} = \frac{250 \text{ V} - 187.5 \text{ V}}{700 \text{ A}} = 0.0893 \Omega$$

This total resistance will be in the circuit until $I_A$ again falls to 350 A. This occurs when $E_A$ reaches

$$E_A = V_T - I_A R_{\text{tot}} = 250 \text{ V} - (350 \text{ A})(0.0893 \Omega) = 218.75 \text{ V}$$

When $E_A = 218.75 \text{ V}$, $I_A$ has fallen to 350 A and it is time to cut out the third starting resistor $R_3$. When it is cut out, only the internal resistance of the motor is left. By now, though, $R_A$ alone can limit the motor's current to

$$I_A = \frac{V_T - E_A}{R_A} = \frac{250 \text{ V} - 218.75 \text{ V}}{0.05 \text{ } \Omega} = 625 \text{ A} \quad (\text{less than allowed maximum})$$

From this point on, the motor can speed up by itself.

From Equations (9-34) to (9-36), the required resistor values can be calculated:

$$R_3 = R_{\text{tot},3} - R_A = 0.0893 \Omega - 0.05 \Omega = 0.0393 \Omega$$
$$R_2 = R_{\text{tot},2} - R_3 - R_A = 0.1786 \Omega - 0.0393 \Omega - 0.05 \Omega = 0.0893 \Omega$$
$$R_1 = R_{\text{tot},1} - R_2 - R_3 - R_A = 0.357 \Omega - 0.1786 \Omega - 0.0393 \Omega - 0.05 \Omega = 0.1786 \Omega$$

And $R_1$, $R_2$, and $R_3$ are cut out when $E_A$ reaches 125, 187.5, and 218.75 V, respectively.

**DC Motor Starting Circuits**

Once the starting resistances have been selected, how can their shorting contacts be controlled to ensure that they shut at exactly the correct moment? Several different schemes are used to accomplish this switching, and two of the most common approaches will be examined in this section. Before that is done, though, it is necessary to introduce some of the components used in motor-starting circuits.

Figure 9–30 illustrates some of the devices commonly used in motor-control circuits. The devices illustrated are fuses, push button switches, relays, time delay relays, and overloads.

Figure 9–30a shows a symbol for a fuse. The fuses in a motor-control circuit serve to protect the motor against the danger of short circuits. They are placed in the power supply lines leading to motors. If a motor develops a short circuit, the fuses in the line leading to it will burn out, opening the circuit before any damage has been done to the motor itself.

Figure 9–30b shows spring-type push button switches. There are two basic types of such switches—normally open and normally shut. *Normally open* contacts are open when the button is resting and closed when the button has been
pushed, while *normally closed* contacts are closed when the button is resting and open when the button has been pushed.

A relay is shown in Figure 9–30c. It consists of a main coil and a number of contacts. The main coil is symbolized by a circle, and the contacts are shown as parallel lines. The contacts are of two types—normally open and normally closed. A *normally open* contact is one which is open when the relay is deenergized, and a *normally closed* contact is one which is closed when the relay is deenergized. When electric power is applied to the relay (the relay is energized), its contacts change state: The normally open contacts close, and the normally closed contacts open.

A time delay relay is shown in Figure 9–30d. It behaves exactly like an ordinary relay except that when it is energized there is an adjustable time delay before its contacts change state.

An overload is shown in Figure 9–30e. It consists of a heater coil and some normally shut contacts. The current flowing to a motor passes through the heater coils. If the load on a motor becomes too large, then the current flowing to the motor will heat up the heater coils, which will cause the normally shut contacts of the overload to open. These contacts can in turn activate some types of motor protection circuitry.

One common motor-starting circuit using these components is shown in Figure 9–31. In this circuit, a series of time delay relays shut contacts which remove each section of the starting resistor at approximately the correct time after power is

---

**FIGURE 9–30**

(a) A fuse. (b) Normally open and normally closed push button switches. (c) A relay coil and contacts. (d) A time delay relay and contacts. (e) An overload and its normally closed contacts.
applied to the motor. When the start button is pushed in this circuit, the motor's armature circuit is connected to its power supply, and the machine starts with all resistance in the circuit. However, relay 1TD energizes at the same time as the motor starts, so after some delay the 1TD contacts will shut and remove part of the starting resistance from the circuit. Simultaneously, relay 2TD is energized, so after another time delay the 2TD contacts will shut and remove the second part of the timing resistor. When the 2TD contacts shut, the 3TD relay is energized, so the process repeats again, and finally the motor runs at full speed with no starting resistance present in its circuit. If the time delays are picked properly, the starting resistors can be cut out at just the right times to limit the motor’s current to its design values.
Another type of motor starter is shown in Figure 9–32. Here, a series of relays sense the value of $E_A$ in the motor and cut out the starting resistance as $E_A$ rises to preset levels. This type of starter is better than the previous one, since if the motor is loaded heavily and starts more slowly than normal, its armature resistance is still cut out when its current falls to the proper value.

Notice that both starter circuits have a relay in the field circuit labeled FL. This is a field loss relay. If the field current is lost for any reason, the field loss
relay is deenergized, which turns off power to the M relay. When the M relay deenergizes, its normally open contacts open and disconnect the motor from the power supply. This relay prevents the motor from running away if its field current is lost.

Notice also that there is an overload in each motor-starter circuit. If the power drawn from the motor becomes excessive, these overloads will heat up and open the OL normally shut contacts, thus turning off the M relay. When the M relay deenergizes, its normally open contacts open and disconnect the motor from the power supply, so the motor is protected against damage from prolonged excessive loads.

9.9 THE WARD-LEONARD SYSTEM AND SOLID-STATE SPEED CONTROLLERS

The speed of a separately excited, shunt, or compounded dc motor can be varied in one of three ways: by changing the field resistance, changing the armature voltage, or changing the armature resistance. Of these methods, perhaps the most useful is armature voltage control, since it permits wide speed variations without affecting the motor's maximum torque.

A number of motor-control systems have been developed over the years to take advantage of the high torques and variable speeds available from the armature voltage control of dc motors. In the days before solid-state electronic components became available, it was difficult to produce a varying dc voltage. In fact, the normal way to vary the armature voltage of a dc motor was to provide it with its own separate dc generator.

An armature voltage control system of this sort is shown in Figure 9–33. This figure shows an ac motor serving as a prime mover for a dc generator, which
in turn is used to supply a dc voltage to a dc motor. Such a system of machines is called a Ward-Leonard system, and it is extremely versatile.

In such a motor-control system, the armature voltage of the motor can be controlled by varying the field current of the dc generator. This armature voltage
allows the motor's speed to be smoothly varied between a very small value and the base speed. The speed of the motor can be adjusted above the base speed by reducing the motor's field current. With such a flexible arrangement, total motor speed control is possible.

Furthermore, if the field current of the generator is reversed, then the polarity of the generator's armature voltage will be reversed, too. This will reverse the motor's direction of rotation. Therefore, it is possible to get a very wide range of speed variations in either direction of rotation out of a Ward-Leonard dc motor-control system.

Another advantage of the Ward-Leonard system is that it can "regenerate," or return the machine's energy of motion to the supply lines. If a heavy load is first raised and then lowered by the dc motor of a Ward-Leonard system, when the load is falling, the dc motor acts as a generator, supplying power back to the power system. In this fashion, much of the energy required to lift the load in the first place can be recovered, reducing the machine's overall operating costs.

The possible modes of operation of the dc machine are shown in the torque-speed diagram in Figure 9-34. When this motor is rotating in its normal direction and supplying a torque in the direction of rotation, it is operating in the first quadrant of this figure. If the generator’s field current is reversed, that will reverse the terminal voltage of the generator, in turn reversing the motor’s armature voltage. When the armature voltage reverses with the motor field current remaining unchanged, both the torque and the speed of the motor are reversed, and the machine is operating as a motor in the third quadrant of the diagram. If the torque or the speed alone of the motor reverses while the other quantity does not, then the machine serves as a generator, returning power to the dc power system. Because

![Torque-speed curves diagram](image-url)
a Ward-Leonard system permits rotation and regeneration in either direction, it is called a four-quadrant control system.

The disadvantages of a Ward-Leonard system should be obvious. One is that the user is forced to buy three full machines of essentially equal ratings, which is quite expensive. Another is that three machines will be much less efficient than one. Because of its expense and relatively low efficiency, the Ward-Leonard system has been replaced in new applications by SCR-based controller circuits.

A simple dc armature voltage controller circuit is shown in Figure 9–35. The average voltage applied to the armature of the motor, and therefore the average speed of the motor, depends on the fraction of the time the supply voltage is applied to the armature. This in turn depends on the relative phase at which the

FIGURE 9–35
(a) A two-quadrant solid-state dc motor controller. Since current cannot flow out of the positive terminals of the armature, this motor cannot act as a generator, returning power to the system.
(b) The possible operating quadrants of this motor controller.
SCRs in the rectifier circuit are triggered. This particular circuit is only capable of supplying an armature voltage with one polarity, so the motor can only be reversed by switching the polarity of its field connection. Notice that it is not possible for an armature current to flow out the positive terminal of this motor, since current cannot flow backward through an SCR. Therefore, this motor cannot regenerate, and any energy supplied to the motor cannot be recovered. This type of control circuit is a two-quadrant controller, as shown in Figure 9–35b.

A more advanced circuit capable of supplying an armature voltage with either polarity is shown in Figure 9–36. This armature voltage control circuit can
permit a current flow out of the positive terminals of the generator, so a motor with this type of controller can regenerate. If the polarity of the motor field circuit can be switched as well, then the solid-state circuit is a full four-quadrant controller like the Ward-Leonard system.

A two-quadrant or a full four-quadrant controller built with SCRs is cheaper than the two extra complete machines needed for the Ward-Leonard system, so solid-state speed-control systems have largely displaced Ward-Leonard systems in new applications.

A typical two-quadrant shunt dc motor drive with armature voltage speed control is shown in Figure 9–37, and a simplified block diagram of the drive is shown in Figure 9–38. This drive has a constant field voltage supplied by a three-phase full-wave rectifier, and a variable armature terminal voltage supplied by six SCRs arranged as a three-phase full-wave rectifier. The voltage supplied to the armature of the motor is controlled by adjusting the firing angle of the SCRs in the bridge. Since this motor controller has a fixed field voltage and a variable armature voltage, it is only able to control the speed of the motor at speeds less than or equal to the base speed (see "Changing the Armature Voltage" in Section 9.4). The controller circuit is identical with that shown in Figure 9–35, except that all of the control electronics and feedback circuits are shown.
A simplified block diagram of the typical solid-state shunt dc motor drive shown in Figure 9-37. (Simplified from a block diagram provided by MagneTek, Inc.)
The major sections of this dc motor drive include:

1. A protection circuit section to protect the motor from excessive armature currents, low terminal voltage, and loss of field current.
2. A start/stop circuit to connect and disconnect the motor from the line.
3. A high-power electronics section to convert three-phase ac power to dc power for the motor's armature and field circuits.
4. A low-power electronics section to provide firing pulses to the SCRs which supply the armature voltage to the motor. This section contains several major subsections, which will be described below.

Protection Circuit Section

The protection circuit section combines several different devices which together ensure the safe operation of the motor. Some typical safety devices included in this type of drive are

1. *Current-limiting fuses*, to disconnect the motor quickly and safely from the power line in the event of a short circuit within the motor. Current-limiting fuses can interrupt currents of up to several hundred thousand amperes.
2. An *instantaneous static trip*, which shuts down the motor if the armature current exceeds 300 percent of its rated value. If the armature current exceeds the maximum allowed value, the trip circuit activates the fault relay, which deenergizes the run relay, opening the main contactors and disconnecting the motor from the line.
3. An *inverse-time overload trip*, which guards against sustained overcurrent conditions not great enough to trigger the instantaneous static trip but large enough to damage the motor if allowed to continue indefinitely. The term *inverse time* implies that the higher the overcurrent flowing in the motor, the faster the overload acts (Figure 9-39). For example, an inverse-time trip might take a full minute to trip if the current flow were 150 percent of the rated current of the motor, but take 10 seconds to trip if the current flow were 200 percent of the rated current of the motor.
4. An *undervoltage trip*, which shuts down the motor if the line voltage supplying the motor drops by more than 20 percent.
5. A *field loss trip*, which shuts down the motor if the field circuit is lost.
6. An *overtemperature trip*, which shuts down the motor if it is in danger of overheating.

Start/Stop Circuit Section

The start/stop circuit section contains the controls needed to start and stop the motor by opening or closing the main contacts connecting the motor to the line. The motor is started by pushing the run button, and it is stopped either by pushing the
stop button or by energizing the fault relay. In either case, the run relay is deenergized, and the main contacts connecting the motor to the line are opened.

**High-Power Electronics Section**

The high-power electronics section contains a three-phase full-wave diode rectifier to provide a constant voltage to the field circuit of the motor and a three-phase full-wave SCR rectifier to provide a variable voltage to the armature circuit of the motor.

**Low-Power Electronics Section**

The low-power electronics section provides firing pulses to the SCRs which supply the armature voltage to the motor. By adjusting the firing time of the SCRs, the low-power electronics section adjusts the motor's average armature voltage. The low-power electronics section contains the following subsystems:

1. *Speed regulation circuit.* This circuit measures the speed of the motor with a tachometer, compares that speed with the desired speed (a reference voltage level), and increases or decreases the armature voltage as necessary to keep the speed constant at the desired value. For example, suppose that the load on the shaft of the motor is increased. If the load is increased, then the motor will slow down. The decrease in speed will reduce the voltage generated by the tachometer, which is fed into the speed regulation circuit. Because the voltage level corresponding to the speed of the motor has fallen below the reference voltage, the speed regulator circuit will advance the firing time of the SCRs, producing a higher armature voltage. The higher armature voltage will tend to increase the speed of the motor back to the desired level (see Figure 9–40).
FIGURE 9-40
(a) The speed regulator circuit produces an output voltage which is proportional to the difference between the desired speed of the motor (set by $V_{\text{ref}}$) and the actual speed of the motor (measured by $V_{\text{tach}}$). This output voltage is applied to the firing circuit in such a way that the larger the output voltage becomes, the earlier the SCRs in the drive turn on and the higher the average terminal voltage becomes. (b) The effect of increasing load on a shunt dc motor with a speed regulator. The load in the motor is increased. If no regulator were present, the motor would slow down and operate at point 2. When the speed regulator is present, it detects the decrease in speed and boosts the armature voltage of the motor to compensate. This raises the whole torque-speed characteristic curve of the motor, resulting in operation at point $2'$. With proper design, a circuit of this type can provide speed regulations of 0.1 percent between no-load and full-load conditions.

The desired operating speed of the motor is controlled by changing the reference voltage level. The reference voltage level can be adjusted with a small potentiometer, as shown in Figure 9-40.
2. **Current-limiting circuit.** This circuit measures the steady-state current flowing to the motor, compares that current with the desired maximum current (set by a reference voltage level), and decreases the armature voltage as necessary to keep the current from exceeding the desired maximum value. The desired maximum current can be adjusted over a wide range, say from 0 to 200 percent or more of the motor’s rated current. This current limit should typically be set at greater than rated current, so that the motor can accelerate under full-load conditions.

3. **Acceleration/deceleration circuit.** This circuit limits the acceleration and deceleration of the motor to a safe value. Whenever a dramatic speed change is commanded, this circuit intervenes to ensure that the transition from the original speed to the new speed is smooth and does not cause an excessive armature current transient in the motor.

The acceleration/deceleration circuit completely eliminates the need for a starting resistor, since starting the motor is just another kind of large speed change, and the acceleration/deceleration circuit acts to cause a smooth increase in speed over time. This gradual smooth increase in speed limits the current flowing in the machine’s armature to a safe value.

### 9.10 DC MOTOR EFFICIENCY CALCULATIONS

To calculate the efficiency of a dc motor, the following losses must be determined:

1. Copper losses
2. Brush drop losses
3. Mechanical losses
4. Core losses
5. Stray losses

The copper losses in the motor are the $IR$ losses in the armature and field circuits of the motor. These losses can be found from a knowledge of the currents in the machine and the two resistances. To determine the resistance of the armature circuit in a machine, block its rotor so that it cannot turn and apply a small dc voltage to the armature terminals. Adjust that voltage until the current flowing in the armature is equal to the rated armature current of the machine. The ratio of the applied voltage to the resulting armature current flow is $R_A$. The reason that the current should be about equal to full-load value when this test is done is that $R_A$ varies with temperature, and at the full-load value of the current, the armature windings will be near their normal operating temperature.

The resulting resistance will not be entirely accurate, because

1. The cooling that normally occurs when the motor is spinning will not be present.
2. Since there is an ac voltage in the rotor conductors during normal operation, they suffer from some amount of skin effect, which further raises armature resistance.

IEEE Standard 113 (Reference 5) deals with test procedures for dc machines. It gives a more accurate procedure for determining $R_A$, which can be used if needed.

The field resistance is determined by supplying the full-rated field voltage to the field circuit and measuring the resulting field current. The field resistance $R_F$ is just the ratio of the field voltage to the field current.

Brush drop losses are often approximately lumped together with copper losses. If they are treated separately, they can be determined from a plot of contact potential versus current for the particular type of brush being used. The brush drop losses are just the product of the brush voltage drop $V_{BD}$ and the armature current $I_A$.

The core and mechanical losses are usually determined together. If a motor is allowed to turn freely at no load and at rated speed, then there is no output power from the machine. Since the motor is at no load, $I_A$ is very small and the armature copper losses are negligible. Therefore, if the field copper losses are subtracted from the input power of the motor, the remaining input power must consist of the mechanical and core losses of the machine at that speed. These losses are called the *no-load rotational losses* of the motor. As long as the motor's speed remains nearly the same as it was when the losses were measured, the no-load rotational losses are a good estimate of mechanical and core losses under load in the machine.

An example of the determination of a motor's efficiency is given below.

**Example 9-8.** A 50-hp, 250-V, 1200 r/min shunt dc motor has a rated armature current of 170 A and a rated field current of 5 A. When its rotor is blocked, an armature voltage of 10.2 V (exclusive of brushes) produces 170 A of current flow, and a field voltage of 250 V produces a field current flow of 5 A. The brush voltage drop is assumed to be 2 V. At no load with the terminal voltage equal to 240 V, the armature current is equal to 13.2 A, the field current is 4.8 A, and the motor's speed is 1150 r/min.

(a) How much power is output from this motor at rated conditions?

(b) What is the motor's efficiency?

**Solution**

The armature resistance of this machine is approximately

$$R_A = \frac{10.2 \text{ V}}{170 \text{ A}} = 0.06 \Omega$$

and the field resistance is

$$R_F = \frac{250 \text{ V}}{5 \text{ A}} = 50 \Omega$$

Therefore, at full load the armature $I^2R$ losses are

$$P_A = (170 \text{ A})^2(0.06 \Omega) = 1734 \text{ W}$$

and the field circuit $I^2R$ losses are
\[ P_F = (5 \text{ A})^2(50 \text{ } \Omega) = 1250 \text{ W} \]

The brush losses at full load are given by
\[ P_{\text{brush}} = V_B D I_A = (2 \text{ V})(170 \text{ A}) = 340 \text{ W} \]

The rotational losses at full load are essentially equivalent to the rotational losses at no load, since the no-load and full-load speeds of the motor do not differ too greatly. These losses may be ascertained by determining the input power to the armature circuit at no load and assuming that the armature copper and brush drop losses are negligible, meaning that the no-load armature input power is equal to the rotational losses:
\[ P_{\text{tot}} = P_{\text{core}} + P_{\text{mech}} = (240 \text{ V})(13.2 \text{ A}) = 3168 \text{ W} \]

(a) The input power of this motor at the rated load is given by
\[ P_{\text{in}} = V_T I_L = (250 \text{ V})(175 \text{ A}) = 43,750 \text{ W} \]

Its output power is given by
\[ P_{\text{out}} = P_{\text{in}} - P_{\text{brush}} - P_{\text{cu}} - P_{\text{core}} - P_{\text{mech}} - P_{\text{stray}} \]
\[ = 43,750 \text{ W} - 340 \text{ W} - 1734 \text{ W} - 1250 \text{ W} - 3168 \text{ W} - (0.01)(43,750 \text{ W}) \]
\[ = 36,820 \text{ W} \]

where the stray losses are taken to be 1 percent of the input power.

(b) The efficiency of this motor at full load is
\[ \eta = \frac{P_{\text{out}}}{P_{\text{out}}} \times 100\% \]
\[ = \frac{36,820 \text{ W}}{43,750 \text{ W}} \times 100\% = 84.2\% \]
10.4 SPEED CONTROL OF SINGLE-PHASE INDUCTION MOTORS

In general, the speed of single-phase induction motors may be controlled in the same manner as the speed of polyphase induction motors. For squirrel-cage rotor motors, the following techniques are available:

1. Vary the stator frequency.
2. Change the number of poles.
3. Change the applied terminal voltage $V_T$.

In practical designs involving fairly high-slip motors, the usual approach to speed control is to vary the terminal voltage of the motor. The voltage applied to a motor may be varied in one of three ways:

1. An autotransformer may be used to continually adjust the line voltage. This is the most expensive method of voltage speed control and is used only when very smooth speed control is needed.
2. An SCR or TRIAC circuit may be used to reduce the rms voltage applied to the motor by ac phase control. This approach chops up the ac waveform as described in Chapter 3 and somewhat increases the motor’s noise and vibration. Solid-state control circuits are considerably cheaper than autotransformers and so are becoming more and more common.
3. A resistor may be inserted in series with the motor’s stator circuit. This is the cheapest method of voltage control, but it has the disadvantage that considerable power is lost in the resistor, reducing the overall power conversion efficiency.

Another technique is also used with very high-slip motors such as shaded-pole motors. Instead of using a separate autotransformer to vary the voltage applied to the stator of the motor, the stator winding itself can be used as an autotransformer. Figure 10–25 shows a schematic representation of a main stator winding, with a number of taps along its length. Since the stator winding is wrapped about an iron core, it behaves as an autotransformer.

When the full line voltage $V$ is applied across the entire main winding, then the induction motor operates normally. Suppose instead that the full line voltage is applied to tap 2, the center tap of the winding. Then an identical voltage will be induced in the upper half of the winding by transformer action, and the total winding...
The use of a stator winding as an autotransformer. If voltage \( V \) is applied to the winding at the center tap, the total winding voltage will be \( 2V \).

**FIGURE 10–26**
The torque-speed characteristic of a shaded-pole induction motor as the terminal voltage is changed. Increases in \( V_T \) may be accomplished either by actually raising the voltage across the whole winding or by switching to a lower tap on the stator winding.

Voltage will be twice the applied line voltage. The total voltage applied to the winding has effectively been doubled.

Therefore, the smaller the fraction of the total coil that the line voltage is applied across, the greater the total voltage will be across the whole winding, and the higher the speed of the motor will be for a given load (see Figure 10–26).

This is the standard approach used to control the speed of single-phase motors in many fan and blower applications. Such speed control has the advantage that it is quite inexpensive, since the only components necessary are taps on the main motor winding and an ordinary multiposition switch. It also has the advantage that the autotransformer effect does not consume power the way series resistors would.
Almost all electric power generation and most of the power transmission in the world today is in the form of three-phase ac circuits. A three-phase ac power system consists of three-phase generators, transmission lines, and loads. AC power systems have a great advantage over dc systems in that their voltage levels can be changed with transformers to reduce transmission losses, as described in Chapter 2. Three-phase ac power systems have two major advantages over single-phase ac power systems: (1) it is possible to get more power per kilogram of metal from a three-phase machine and (2) the power delivered to a three-phase load is constant at all times, instead of pulsing as it does in single-phase systems. Three-phase systems also make the use of induction motors easier by allowing them to start without special auxiliary starting windings.

A.1 GENERATION OF THREE-PHASE VOLTAGES AND CURRENTS

A three-phase generator consists of three single-phase generators, with voltages equal in magnitude but differing in phase angle from the others by 120°. Each of these three generators could be connected to one of three identical loads by a pair of wires, and the resulting power system would be as shown in Figure A–1c. Such a system consists of three single-phase circuits that happen to differ in phase angle by 120°. The current flowing to each load can be found from the equation

\[ I = \frac{V}{Z} \]  

(A–1)
(a) A three-phase generator, consisting of three single-phase sources equal in magnitude and 120° apart in phase. (b) The voltages in each phase of the generator. (c) The three phases of the generator connected to three identical loads.
FIGURE A-1 (concluded)
(d) Phasor diagram showing the voltages in each phase.

\[ V_A, V_B, V_C \]

FIGURE A-2
The three circuits connected together with a common neutral.

Therefore, the currents flowing in the three phases are

\[ I_A = \frac{V \angle 0^\circ}{Z \angle \theta} = I \angle -\theta \quad (A-2) \]
\[ I_B = \frac{V \angle -120^\circ}{Z \angle \theta} = I \angle -120^\circ - \theta \quad (A-3) \]
\[ I_C = \frac{V \angle -240^\circ}{Z \angle \theta} = I \angle -240^\circ - \theta \quad (A-4) \]

It is possible to connect the negative ends of these three single-phase generators and loads together, so that they share a common return line (called the neutral). The resulting system is shown in Figure A-2; note that now only four wires are required to supply power from the three generators to the three loads.

How much current is flowing in the single neutral wire shown in Figure A-2? The return current will be the sum of the currents flowing to each individual load in the power system. This current is given by
\[ I_N = I_A + I_B + I_C \]  
\[ = I \angle -\theta + I \angle -\theta - 120^\circ + I \angle -\theta - 240^\circ \]
\[ = I \cos (-\theta) + jI \sin (-\theta) \]
\[ + I \cos (-\theta - 120^\circ) + jI \sin (-\theta - 120^\circ) \]
\[ + I \cos (-\theta - 240^\circ) + jI \sin (-\theta - 240^\circ) \]
\[ = I [\cos (-\theta) + \cos (-\theta - 120^\circ) + \cos (-\theta - 240^\circ)] \]
\[ + jI [\sin (-\theta) + \sin (-\theta - 120^\circ) + \sin (-\theta - 240^\circ)] \]

Recall the elementary trigonometric identities:
\[
\cos (\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \quad (A-6)
\]
\[
\sin (\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \quad (A-7)
\]

Applying these trigonometric identities yields
\[
I_N = I \cos (-\theta) + \cos (-\theta) \cos 120^\circ + \sin (-\theta) \sin 120^\circ + \cos (-\theta) \cos 240^\circ \\
+ \sin (-\theta) \sin 240^\circ]
\]
\[
+ jI [\sin (-\theta) + \sin (-\theta) \cos 120^\circ - \cos (-\theta) \sin 120^\circ \\
+ \sin (-\theta) \cos 240^\circ - \cos (-\theta) \sin 240^\circ] 
\]
\[
I_N = I \left[ \cos (-\theta) - \frac{1}{2} \cos (-\theta) + \frac{\sqrt{3}}{2} \sin (-\theta) - \frac{1}{2} \cos (-\theta) - \frac{\sqrt{3}}{2} \sin (-\theta) \right] 
\]
\[
+ jI \left[ \sin (-\theta) - \frac{1}{2} \sin (-\theta) - \frac{\sqrt{3}}{2} \cos (-\theta) - \frac{1}{2} \sin (-\theta) + \frac{\sqrt{3}}{2} \cos (-\theta) \right] 
\]
\[
I_N = 0 \text{ A} 
\]

As long as the three loads are equal, the return current in the neutral is zero! A three-phase power system in which the three generators have voltages that are exactly equal in magnitude and 120° different in phase, and in which all three loads are identical, is called a balanced three-phase system. In such a system, the neutral is actually unnecessary, and we could get by with only three wires instead of the original six.

PHASE SEQUENCE. The phase sequence of a three-phase power system is the order in which the voltages in the individual phases peak. The three-phase power system illustrated in Figure A-1 is said to have phase sequence abc, since the voltages in the three phases peak in the order a, b, c (see Figure A-1b). The phasor diagram of a power system with an abc phase sequence is shown in Figure A-3a.

It is also possible to connect the three phases of a power system so that the voltages in the phases peak in order the order a, c, b. This type of power system is said to have phase sequence acb. The phasor diagram of a power system with an acb phase sequence is shown in Figure A-3b.

The result derived above is equally valid for both abc and acb phase sequences. In either case, if the power system is balanced, the current flowing in the neutral will be 0.
A.2 VOLTAGES AND CURRENTS IN A THREE-PHASE CIRCUIT

A connection of the sort shown in Figure A–2 is called a wye (Y) connection because it looks like the letter Y. Another possible connection is the delta (Δ) connection, in which the three generators are connected head to tail. The Δ connection is possible because the sum of the three voltages \( V_A + V_B + V_C = 0 \), so that no short-circuit currents will flow when the three sources are connected head to tail.

Each generator and each load in a three-phase power system may be either Y- or Δ-connected. Any number of Y- and Δ-connected generators and loads may be mixed on a power system.

Figure A–4 shows three-phase generators connected in Y and in Δ. The voltages and currents in a given phase are called phase quantities, and the voltages between lines and currents in the lines connected to the generators are called line quantities. The relationship between the line quantities and phase quantities for a given generator or load depends on the type of connection used for that generator or load. These relationships will now be explored for each of the Y and Δ connections.

### Voltages and Currents in the Wye (Y) Connection

A Y-connected three-phase generator with an abc phase sequence connected to a resistive load is shown in Figure A–5. The phase voltages in this generator are given by

\[
\begin{align*}
V_{an} &= V_a \angle 0^\circ \\
V_{bn} &= V_b \angle -120^\circ \\
V_{cn} &= V_c \angle -240^\circ
\end{align*}
\]  

(A–8)
Since the load connected to this generator is assumed to be resistive, the current in each phase of the generator will be at the same angle as the voltage. Therefore, the current in each phase will be given by

\[
\begin{align*}
I_a &= I_\phi \angle 0^\circ \\
I_b &= I_\phi \angle -120^\circ \\
I_c &= I_\phi \angle -240^\circ
\end{align*}
\]  

(A-9)
From Figure A–5, it is obvious that the current in any line is the same as the current in the corresponding phase. Therefore, for a Y connection,

\[ I_L = I_\phi \quad \text{Y connection} \] (A-10)

The relationship between line voltage and phase voltage is a bit more complex. By Kirchhoff’s voltage law, the line-to-line voltage \( V_{ab} \) is given by

\[
V_{ab} = V_a - V_b \\
= V_\phi \angle 0^\circ - V_\phi \angle -120^\circ \\
= V_\phi - \left( -\frac{1}{2} V_\phi - j \frac{\sqrt{3}}{2} V_\phi \right) = \frac{3}{2} V_\phi + j \frac{\sqrt{3}}{2} V_\phi \\
= \sqrt{3} V_\phi \left( \frac{\sqrt{3}}{2} + j \frac{1}{2} \right) \\
= \sqrt{3} V_\phi \angle 30^\circ
\]

Therefore, the relationship between the magnitudes of the line-to-line voltage and the line-to-neutral (phase) voltage in a Y-connected generator or load is

\[ V_{LL} = \sqrt{3} V_\phi \quad \text{Y connection} \] (A-11)

In addition, the line voltages are shifted 30° with respect to the phase voltages. A phasor diagram of the line and phase voltages for the Y connection in Figure A–5 is shown in Figure A–6.

Note that for Y connections with the \( abc \) phase sequence such as the one in Figure A–5, the voltage of a line leads the corresponding phase voltage by 30°. For Y connections with the \( acb \) phase sequence, the voltage of a line lags the corresponding phase voltage by 30°, as you will be asked to demonstrate in a problem at the end of the appendix.
Although the relationships between line and phase voltages and currents for the Y connection were derived for the assumption of a unity power factor, they are in fact valid for any power factor. The assumption of unity-power-factor loads simply made the mathematics slightly easier in this development.

**Voltages and Currents in the Delta (Δ) Connection**

A Δ-connected three-phase generator connected to a resistive load is shown in Figure A–7. The phase voltages in this generator are given by

\[
\begin{align*}
V_{ab} &= V_\phi \angle 0^\circ \\
V_{bc} &= V_\phi \angle -120^\circ \\
V_{ca} &= V_\phi \angle -240^\circ
\end{align*}
\]  
(A–12)

Because the load is resistive, the phase currents are given by

\[
\begin{align*}
I_{ab} &= I_\phi \angle 0^\circ \\
I_{bc} &= I_\phi \angle -120^\circ \\
I_{ca} &= I_\phi \angle -240^\circ
\end{align*}
\]  
(A–13)

In the case of the Δ connection, it is obvious that the line-to-line voltage between any two lines will be the same as the voltage in the corresponding phase. In a Δ connection,

\[
V_{LL} = V_\phi \quad \text{Δ connection}
\]  
(A–14)

The relationship between line current and phase current is more complex. It can be found by applying Kirchhoff’s current law at a node of the Δ. Applying Kirchhoff’s current law to node A yields the equation

\[
I_a = I_{ab} - I_{ca} \\
= I_\phi \angle 0^\circ - I_\phi \angle -240^\circ \\
= I_\phi - \left( -\frac{1}{2} I_\phi + j \frac{\sqrt{3}}{2} I_\phi \right) = \frac{3}{2} I_\phi - j \frac{\sqrt{3}}{2} I_\phi
\]
Therefore, the relationship between the magnitudes of the line and phase currents in a Δ-connected generator or load is

\[ I_L = \sqrt{3} I_\phi \quad \Delta \text{ connection} \]  

and the line currents are shifted 30° relative to the corresponding phase currents.

Note that for Δ connections with the abc phase sequence such as the one shown in Figure A-7, the current of a line lags the corresponding phase current by 30° (see Figure A-8). For Δ connections with the acb phase sequence, the current of a line leads the corresponding phase current by 30°.

The voltage and current relationships for Y- and Δ-connected sources and loads are summarized in Table A-1.
A.3 POWER RELATIONSHIPS IN THREE-PHASE CIRCUITS

Figure A–9 shows a balanced Y-connected load whose phase impedance is \( Z_\phi = Z \angle \theta \). If the three-phase voltages applied to this load are given by

\[
\begin{align*}
    v_{an}(t) &= \sqrt{2}V \sin \omega t \\
    v_{bn}(t) &= \sqrt{2}V \sin(\omega t - 120^\circ) \\
    v_{cn}(t) &= \sqrt{2}V \sin(\omega t - 240^\circ)
\end{align*}
\]

then the three-phase currents flowing in the load are given by

\[
\begin{align*}
    i_a(t) &= \sqrt{2}I \sin(\omega t - \theta) \\
    i_b(t) &= \sqrt{2}I \sin(\omega t - 120^\circ - \theta) \\
    i_c(t) &= \sqrt{2}I \sin(\omega t - 240^\circ - \theta)
\end{align*}
\]

where \( I = V/Z \). How much power is being supplied to this load from the source?

The instantaneous power supplied to one phase of the load is given by the equation

\[ p(t) = v(t)i(t) \]  

(A–18)

Therefore, the instantaneous power supplied to each of the three phases is

\[
\begin{align*}
    p_a(t) &= v_{an}(t)i_a(t) = 2VI \sin(\omega t) \sin(\omega t - \theta) \\
    p_b(t) &= v_{bn}(t)i_b(t) = 2VI \sin(\omega t - 120^\circ) \sin(\omega t - 120^\circ - \theta) \\
    p_c(t) &= v_{cn}(t)i_c(t) = 2VI \sin(\omega t - 240^\circ) \sin(\omega t - 240^\circ - \theta)
\end{align*}
\]

(A–19)

A trigonometric identity states that

\[
\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha - \beta)]
\]

(A–20)

Applying this identity to Equations (A–19) yields new expressions for the power in each phase of the load:
Instantaneous power in phases $a$, $b$, and $c$, plus the total power supplied to the load.

\[
\begin{align*}
    p_a(t) &= V_I[\cos \theta - \cos(2\omega t - \theta)] \\
    p_b(t) &= V_I[\cos \theta - \cos(2\omega t - 240^\circ - \theta)] \\
    p_c(t) &= V_I[\cos \theta - \cos(2\omega t - 480^\circ - \theta)]
\end{align*}
\]  \hspace{1cm} (A–21)

The total power supplied to the entire three-phase load is the sum of the power supplied to each of the individual phases. The power supplied by each phase consists of a constant component plus a pulsing component. However, the pulsing components in the three phases cancel each other out since they are 120° out of phase with each other, and the final power supplied by the three-phase power system is constant. This power is given by the equation:

\[
p_{\text{tot}}(t) = p_a(t) + p_b(t) + p_c(t) = 3VI \cos \theta
\]  \hspace{1cm} (A–22)

The instantaneous power in phases $a$, $b$, and $c$ are shown as a function of time in Figure A–10. Note that the total power supplied to a balanced three-phase load is constant at all times. The fact that a constant power is supplied by a three-phase power system is one of its major advantages compared to single-phase sources.

**Three-Phase Power Equations Involving Phase Quantities**

The single-phase power Equations (1–60) to (1–66) apply to each phase of a Y- or \( \Delta \)-connected three-phase load, so the real, reactive, and apparent powers supplied to a balanced three-phase load are given by
The angle $\theta$ is again the angle between the voltage and the current in any phase of the load (it is the same in all phases), and the power factor of the load is the cosine of the impedance angle $\theta$. The power-triangle relationships apply as well.

### Three-Phase Power Equations Involving Line Quantities

It is also possible to derive expressions for the power in a balanced three-phase load in terms of line quantities. This derivation must be done separately for Y- and $\Delta$-connected loads, since the relationships between the line and phase quantities are different for each type of connection.

For a Y-connected load, the power consumed by a load is given by

$$ P = 3V_\phi I_\phi \cos \theta $$  \hspace{1cm} (A–23)

For this type of load, $I_L = I_\phi$ and $V_{LL} = \sqrt{3}V_\phi$, so the power consumed by the load can also be expressed as

$$ P = 3 \left( \frac{V_{LL}}{\sqrt{3}} \right) I_L \cos \theta $$

$$ P = \sqrt{3}V_{LL} I_L \cos \theta $$  \hspace{1cm} (A–29)

For a $\Delta$-connected load, the power consumed by a load is given by

$$ P = 3V_\phi I_\phi \cos \theta $$  \hspace{1cm} (A–23)

For this type of load, $I_L = \sqrt{3}I_\phi$ and $V_{LL} = V_\phi$, so the power consumed by the load can also be expressed in terms of line quantities as

$$ P = 3V_{LL} \left( \frac{I_L}{\sqrt{3}} \right) \cos \theta $$

$$ = \sqrt{3}V_{LL} I_L \cos \theta $$  \hspace{1cm} (A–29)
This is exactly the same equation that was derived for a Y-connected load, so Equation (A-29) gives the power of a balanced three-phase load in terms of line quantities regardless of the connection of the load. The reactive and apparent powers of the load in terms of line quantities are

\[ Q = \sqrt{3} V_{LL} I_L \sin \theta \]  \hfill (A-30)

\[ S = \sqrt{3} V_{LL} I_L \]  \hfill (A-31)

It is important to realize that the \( \cos \theta \) and \( \sin \theta \) terms in Equations (A-29) and (A-30) are the cosine and sine of the angle between the phase voltage and the phase current, not the angle between the line-to-line voltage and the line current. Remember that there is a \( 30^\circ \) phase shift between the line-to-line and phase voltage for a Y connection, and between the line and phase current for a \( \Delta \) connection, so it is important not to take the cosine of the angle between the line-to-line voltage and line current.

**A.4 ANALYSIS OF BALANCED THREE-PHASE SYSTEMS**

If a three-phase power system is balanced, it is possible to determine the voltages, currents, and powers at various points in the circuit with a per-phase equivalent circuit. This idea is illustrated in Figure A-11. Figure A-11a shows a Y-connected generator supplying power to a Y-connected load through a three-phase transmission line.

In such a balanced system, a neutral wire may be inserted with no effect on the system, since no current flows in that wire. This system with the extra wire inserted is shown in Figure A-11b. Also, notice that each of the three phases is identical except for a \( 120^\circ \) shift in phase angle. Therefore, it is possible to analyze a circuit consisting of one phase and the neutral, and the results of that analysis will be valid for the other two phases as well if the \( 120^\circ \) phase shift is included. Such a per-phase circuit is shown in Figure A-11c.

There is one problem associated with this approach, however. It requires that a neutral line be available (at least conceptually) to provide a return path for current flow from the loads to the generator. This is fine for Y-connected sources and loads, but no neutral can be connected to \( \Delta \)-connected sources and loads.

How can \( \Delta \)-connected sources and loads be included in a power system to be analyzed? The standard approach is to transform the impedances by the Y–\( \Delta \) transform of elementary circuit theory. For the special case of balanced loads, the Y–\( \Delta \) transformation states that a \( \Delta \)-connected load consisting of three equal impedances, each of value \( Z \), is totally equivalent to a Y-connected load consisting of three impedances, each of value \( Z/3 \) (see Figure A-12). This equivalence means that the voltages, currents, and powers supplied to the two loads cannot be distinguished in any fashion by anything external to the load itself.
(a) A Y-connected generator and load. (b) System with neutral inserted. (c) The per-phase equivalent circuit.
FIGURE A-12
Y-Δ transformation. A Y-connected impedance of Z/3 Ω is totally equivalent to a Δ-connected impedance of Z Ω to any circuit connected to the load’s terminals.

FIGURE A-13
The three-phase circuit of Example A-1.

If Δ-connected sources or loads include voltage sources, then the magnitudes of the voltage sources must be scaled according to Equation (A-11), and the effect of the 30° phase shift must be included as well.

Example A-1. A 208-V three-phase power system is shown in Figure A-13. It consists of an ideal 208-V Y-connected three-phase generator connected through a three-phase transmission line to a Y-connected load. The transmission line has an impedance of 0.06 + j0.12 Ω per phase, and the load has an impedance of 12 + j9 Ω per phase. For this simple power system, find

(a) The magnitude of the line current $I_L$
(b) The magnitude of the load’s line and phase voltages $V_{LL}$ and $V_{ph}$
(c) The real, reactive, and apparent powers consumed by the load

(d) The power factor of the load

(e) The real, reactive, and apparent powers consumed by the transmission line

(f) The real, reactive, and apparent powers supplied by the generator

(g) The generator's power factor

Solution

Since both the generator and the load on this power system are Y-connected, it is very simple to construct a per-phase equivalent circuit. This circuit is shown in Figure A-14.

(a) The line current flowing in the per-phase equivalent circuit is given by

\[ I_{\text{line}} = \frac{V}{Z_{\text{line}} + Z_{\text{load}}} \]

\[ = \frac{120 \angle 0^\circ}{(0.06 + j0.12 \, \Omega) + (12 + j9 \, \Omega)} \]

\[ = \frac{120 \angle 0^\circ}{12.06 + j9.12} = \frac{120 \angle 0^\circ}{15.12 \angle 37.1^\circ} \]

\[ = 7.94 \angle -37.1^\circ \, \text{A} \]

The magnitude of the line current is thus 7.94 A.

(b) The phase voltage on the load is the voltage across one phase of the load. This voltage is the product of the phase impedance and the phase current of the load:

\[ V_{\phi L} = I_{\phi L} Z_{\phi L} \]

\[ = (7.94 \angle -37.1^\circ \, \text{A})(12 + j9 \, \Omega) \]

\[ = (7.94 \angle -37.1^\circ \, \text{A})(15 \angle 36.9^\circ \, \Omega) \]

\[ = 119.1 \angle -0.2^\circ \, \text{V} \]

Therefore, the magnitude of the load's phase voltage is

\[ V_{\phi L} = 119.1 \, \text{V} \]

and the magnitude of the load's line voltage is

\[ V_{LL} = \sqrt{3} V_{\phi L} = 206.3 \, \text{V} \]

(c) The real power consumed by the load is

\[ P_{\text{load}} = 3 V_{\phi} I_{\phi} \cos \theta \]

\[ = 3(119.1 \, \text{V})(7.94 \, \text{A}) \cos 36.9^\circ \]

\[ = 2270 \, \text{W} \]
The reactive power consumed by the load is

\[ Q_{\text{load}} = 3V_I I_0 \sin \theta \]
\[ = 3(119.1 \text{ V})(7.94 \text{ A}) \sin 36.9^\circ \]
\[ = 1702 \text{ var} \]

The apparent power consumed by the load is

\[ S_{\text{load}} = 3V_I I_0 \]
\[ = 3(119.1 \text{ V})(7.94 \text{ A}) \]
\[ = 2839 \text{ VA} \]

\((d)\) The load power factor is

\[ \text{PF}_{\text{load}} = \cos \theta = \cos 36.9^\circ = 0.8 \text{ lagging} \]

\((e)\) The current in the transmission line is 7.94 \(\angle -37.1\) A, and the impedance of the line is 0.06 + j0.12 \(\Omega\) or 0.134 \(\angle 63.4^\circ\) \(\Omega\) per phase. Therefore, the real, reactive, and apparent powers consumed in the line are

\[ P_{\text{line}} = 3I_0^2 Z \cos \theta \]
\[ = 3(7.94 \text{ A})^2 (0.134 \text{ } \Omega) \cos 63.4^\circ \]
\[ = 11.3 \text{ W} \]

\[ Q_{\text{line}} = 3I_0^2 Z \sin \theta \]
\[ = 3(7.94 \text{ A})^2 (0.134 \text{ } \Omega) \sin 63.4^\circ \]
\[ = 22.7 \text{ var} \]

\[ S_{\text{line}} = 3I_0^2 Z \]
\[ = 3(7.94 \text{ A})^2 (0.134 \text{ } \Omega) \]
\[ = 25.3 \text{ VA} \]

\((f)\) The real and reactive powers supplied by the generator are the sum of the powers consumed by the line and the load:

\[ P_{\text{gen}} = P_{\text{line}} + P_{\text{load}} \]
\[ = 11.3 \text{ W} + 2270 \text{ W} = 2281 \text{ W} \]

\[ Q_{\text{gen}} = Q_{\text{line}} + Q_{\text{load}} \]
\[ = 22.7 \text{ var} + 1702 \text{ var} = 1725 \text{ var} \]

The apparent power of the generator is the square root of the sum of the squares of the real and reactive powers:

\[ S_{\text{gen}} = \sqrt{P_{\text{gen}}^2 + Q_{\text{gen}}^2} = 2860 \text{ VA} \]

\((g)\) From the power triangle, the power-factor angle \(\theta\) is

\[ \theta_{\text{gen}} = \tan^{-1} \frac{Q_{\text{gen}}}{P_{\text{gen}}} = \tan^{-1} \frac{1725 \text{ VAR}}{2281 \text{ W}} = 37.1^\circ \]

Therefore, the generator's power factor is

\[ \text{PF}_{\text{gen}} = \cos 37.1^\circ = 0.798 \text{ lagging} \]
Example A–2. Repeat Example A–1 for a Δ-connected load, with everything else unchanged.

Solution
This power system is shown in Figure A–15. Since the load on this power system is Δ connected, it must first be converted to an equivalent Y form. The phase impedance of the Δ-connected load is $12 + j9 \ \Omega$ so the equivalent phase impedance of the corresponding Y form is

$$Z_Y = \frac{Z_\Delta}{3} = 4 + j3 \ \Omega$$

The resulting per-phase equivalent circuit of this system is shown in Figure A–16.

(a) The line current flowing in the per-phase equivalent circuit is given by

$$I_{line} = \frac{V}{Z_{line} + Z_{load}}$$
The magnitude of the line current is thus 23.4 A.

(b) The phase voltage on the equivalent Y load is the voltage across one phase of the load. This voltage is the product of the phase impedance and the phase current of the load:

\[ V_{\phi L} = V_{\phi} Z_{\phi L} \]

\[ = (23.4\angle -37.5^\circ \text{ A})(4 + j3 \Omega) \]

\[ = (23.4\angle -37.5^\circ \text{ A})(5\angle 36.9^\circ \Omega) = 117\angle -0.6^\circ \text{ V} \]

The original load was \( \Delta \) connected, so the phase voltage of the original load is

\[ V_{\phi L} = \sqrt{3} (117 \text{ V}) = 203 \text{ V} \]

and the magnitude of the load's line voltage is

\[ V_{LL} = V_{\phi L} = 203 \text{ V} \]

(c) The real power consumed by the equivalent Y load (which is the same as the power in the actual load) is

\[ P_{\text{load}} = 3V_{\phi} I_{\phi} \cos \theta \]

\[ = 3(117 \text{ V})(23.4 \text{ A}) \cos 36.9^\circ \]

\[ = 6571 \text{ W} \]

The reactive power consumed by the load is

\[ Q_{\text{load}} = 3V_{\phi} I_{\phi} \sin \theta \]

\[ = 3(117 \text{ V})(23.4 \text{ A}) \sin 36.9^\circ \]

\[ = 4928 \text{ var} \]

The apparent power consumed by the load is

\[ S_{\text{load}} = 3V_{\phi} I_{\phi} \]

\[ = 3(117 \text{ V})(23.4 \text{ A}) \]

\[ = 8213 \text{ VA} \]

(d) The load power factor is

\[ \text{PF}_{\text{load}} = \cos \theta = \cos 36.9^\circ = 0.8 \text{ lagging} \]

(e) The current in the transmission is \( 23.4\angle -37.5^\circ \text{ A} \), and the impedance of the line is \( 0.06 + j0.12 \Omega \) or \( 0.134\angle 63.4^\circ \Omega \) per phase. Therefore, the real, reactive, and apparent powers consumed in the line are

\[ P_{\text{line}} = 3I_{\phi}^2 Z \cos \theta \]

\[ = 3(23.4 \text{ A})^2(0.134 \Omega) \cos 63.4^\circ \]

\[ = 98.6 \text{ W} \]
\[ Q_{\text{line}} = 3I_{A}^{2}Z \sin \theta \]  
\[ = 3(23.4 \text{ A})^{2}(0.134 \text{ } \Omega) \sin 63.4^\circ \]  
\[ = 197 \text{ var} \]  
\[ S_{\text{line}} = 3I_{A}^{2}Z \]  
\[ = 3(23.4 \text{ A})^{2}(0.134 \text{ } \Omega) \]  
\[ = 220 \text{ VA} \]  

\[(f)\] The real and reactive powers supplied by the generator are the sums of the powers consumed by the line and the load:

\[ P_{\text{gen}} = P_{\text{line}} + P_{\text{load}} \]  
\[ = 98.6 \text{ W} + 6571 \text{ W} = 6670 \text{ W} \]  
\[ Q_{\text{gen}} = Q_{\text{line}} + Q_{\text{load}} \]  
\[ = 197 \text{ var} + 4928 \text{ VAR} = 5125 \text{ var} \]  

The apparent power of the generator is the square root of the sum of the squares of the real and reactive powers:

\[ S_{\text{gen}} = \sqrt{P_{\text{gen}}^{2} + Q_{\text{gen}}^{2}} = 8411 \text{ VA} \]  

\[(g)\] From the power triangle, the power-factor angle \( \theta \) is

\[ \theta_{\text{gen}} = \tan^{-1} \frac{Q_{\text{gen}}}{P_{\text{gen}}} = \tan^{-1} \frac{5125 \text{ var}}{6670 \text{ W}} = 37.6^\circ \]  

Therefore, the generator's power factor is

\[ \text{PF}_{\text{gen}} = \cos 37.6^\circ = 0.792 \text{ lagging} \]  

### A.5 ONE-LINE DIAGRAMS

As we have seen in this chapter, a balanced three-phase power system has three lines connecting each source with each load, one for each of the phases in the power system. The three phases are all similar, with voltages and currents equal in amplitude and shifted in phase from each other by 120°. Because the three phases are all basically the same, it is customary to sketch power systems in a simple form with a single line representing all three phases of the real power system. These one-line diagrams provide a compact way to represent the interconnections of a power system. One-line diagrams typically include all of the major components of a power system, such as generators, transformers, transmission lines, and loads with the transmission lines represented by a single line. The voltages and types of connections of each generator and load are usually shown on the diagram. A simple power system is shown in Figure A–17, together with the corresponding one-line diagram.

### A.6 USING THE POWER TRIANGLE

If the transmission lines in a power system can be assumed to have negligible impedance, then an important simplification is possible in the calculation of three-
phase currents and powers. This simplification depends on the use of the real and reactive powers of each load to determine the currents and power factors at various points in the system.

For example, consider the simple power system shown in Figure A-17. If the transmission line in that power system is assumed to be lossless, the line voltage at the generator will be the same as the line voltage at the loads. If the generator voltage is specified, then we can find the current and power factor at any point in this power system as follows:

1. Determine the line voltage at the generator and the loads. Since the transmission line is assumed to be lossless, these two voltages will be identical.
2. Determine the real and reactive powers of each load on the power system. We can use the known load voltage to perform this calculation.
3. Find the total real and reactive powers supplied to all loads "downstream" from the point being examined.
4. Determine the system power factor at that point, using the power-triangle relationships.

5. Use Equation (A–29) to determine line currents, or Equation (A–23) to determine phase currents, at that point.

This approach is commonly employed by engineers estimating the currents and power flows at various points on distribution systems within an industrial plant. Within a single plant, the lengths of transmission lines will be quite short and their impedances will be relatively small, and so only small errors will occur if the impedances are neglected. An engineer can treat the line voltage as constant, and use the power triangle method to quickly calculate the effect of adding a load on the overall system current and power factor.

Example A–3. Figure A–18 shows a one-line diagram of a small 480-V industrial distribution system. The power system supplies a constant line voltage of 480 V, and the impedance of the distribution lines is negligible. Load 1 is a Δ-connected load with a phase impedance of $10 \angle 30^\circ \Omega$, and load 2 is a Y-connected load with a phase impedance of $5 \angle -36.87^\circ \Omega$.

(a) Find the overall power factor of the distribution system.
(b) Find the total line current supplied to the distribution system.

Solution
The lines in this system are assumed impedanceless, so there will be no voltage drops within the system. Since load 1 is Δ connected, its phase voltage will be 480 V. Since load 2 is Y connected, its phase voltage will be $480/\sqrt{3} = 277$ V.

The phase current in load 1 is

$$I_{\phi 1} = \frac{480 \text{ V}}{10 \text{ \Omega}} = 48 \text{ A}$$

Therefore, the real and reactive powers of load 1 are

$$P_1 = 3V_{\phi 1}I_{\phi 1} \cos \theta$$
$$= 3(480 \text{ V})(48 \text{ A}) \cos 30^\circ = 59.9 \text{ kW}$$
\[ Q_1 = 3V_\phi I_{\phi 1} \sin \theta \]
\[ = 3(480 \text{ V})(48 \text{ A}) \sin 30^\circ = 34.6 \text{ kvar} \]

The phase current in load 2 is
\[ I_{\phi 2} = \frac{277 \text{ V}}{5 \Omega} = 55.4 \text{ A} \]

Therefore, the real and reactive powers of load 2 are
\[ P_2 = 3V_{\phi 2} I_{\phi 2} \cos \theta \]
\[ = 3(277 \text{ V})(55.4 \text{ A}) \cos(-36.87^\circ) = 36.8 \text{ kW} \]
\[ Q_2 = 3V_{\phi 2} I_{\phi 2} \sin \theta \]
\[ = 3(277 \text{ V})(55.4 \text{ A}) \sin(-36.87^\circ) = -27.6 \text{ kvar} \]

(a) The total real and reactive powers supplied by the distribution system are
\[ P_{\text{tot}} = P_1 + P_2 \]
\[ = 59.9 \text{ kW} + 36.8 \text{ kW} = 96.7 \text{ kW} \]
\[ Q_{\text{tot}} = Q_1 + Q_2 \]
\[ = 34.6 \text{ kvar} - 27.6 \text{ kvar} = 7.00 \text{ kvar} \]

From the power triangle, the effective impedance angle \( \theta \) is given by
\[ \theta = \tan^{-1} \frac{Q}{P} \]
\[ = \tan^{-1} \frac{7.00 \text{ kvar}}{96.7 \text{ kW}} = 4.14^\circ \]

The system power factor is thus
\[ \text{PF} = \cos \theta = \cos(4.14^\circ) = 0.997 \text{ lagging} \]

(b) The total line current is given by
\[ I_L = \frac{P}{\sqrt{3}V_L \cos \theta} \]
\[ = \frac{96.7 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.997)} = 117 \text{ A} \]

**QUESTIONS**

A–1. What types of connections are possible for three-phase generators and loads?
A–2. What is meant by the term "balanced" in a balanced three-phase system?
A–3. What is the relationship between phase and line voltages and currents for a wye (Y) connection?
A–4. What is the relationship between phase and line voltages and currents for a delta (Δ) connection?
A–5. What is phase sequence?
A–6. Write the equations for real, reactive, and apparent power in three-phase circuits, in terms of both line and phase quantities.
A–7. What is a Y–Δ transform?
PROBLEMS

A-1. Three impedances of $4 + j3 \, \Omega$ are $\Delta$ connected and tied to a three-phase 208-V power line. Find $I_\phi$, $I_L$, $P$, $Q$, $S$, and the power factor of this load.

A-2. Figure PA-1 shows a three-phase power system with two loads. The $\Delta$-connected generator is producing a line voltage of 480 V, and the line impedance is $0.09 + j0.16 \, \Omega$. Load 1 is $Y$ connected, with a phase impedance of $2.5\angle36.87^\circ \, \Omega$ and load 2 is $\Delta$ connected, with a phase impedance of $5\angle-20^\circ \, \Omega$.

![Figure PA-1](image)

**FIGURE PA-1**
The system in Problem A-2.

(a) What is the line voltage of the two loads?
(b) What is the voltage drop on the transmission lines?
(c) Find the real and reactive powers supplied to each load.
(d) Find the real and reactive power losses in the transmission line.
(e) Find the real power, reactive power, and power factor supplied by the generator.

A-3. Figure PA-2 shows a one-line diagram of a simple power system containing a single 480-V generator and three loads. Assume that the transmission lines in this power system are lossless, and answer the following questions.

(a) Assume that Load 1 is $Y$ connected. What are the phase voltage and currents in that load?
(b) Assume that Load 2 is $\Delta$ connected. What are the phase voltage and currents in that load?
(c) What real, reactive, and apparent power does the generator supply when the switch is open?
(d) What is the total line current $I_L$ when the switch is open?
(e) What real, reactive, and apparent power does the generator supply when the switch is closed?
THREE-PHASE CIRCUITS

FIGURE PA-2
The power system in Problem A-3.

(f) What is the total line current \( I_L \) when the switch is closed?

(g) How does the total line current \( I_L \) compare to the sum of the three individual currents \( I_1 + I_2 + I_3 \)? If they are not equal, why not?

A-4. Prove that the line voltage of a Y-connected generator with an \( abc \) phase sequence lags the corresponding phase voltage by 30°. Draw a phasor diagram showing the phase and line voltages for this generator.

A-5. Find the magnitudes and angles of each line and phase voltage and current on the load shown in Figure PA-3.

FIGURE PA-3
The system in Problem A-5.

A-6. Figure PA-4 shows a one-line diagram of a small 480-V distribution system in an industrial plant. An engineer working at the plant wishes to calculate the current that will be drawn from the power utility company with and without the capacitor bank switched into the system. For the purposes of this calculation, the engineer will assume that the lines in the system have zero impedance.

(a) If the switch shown is open, find the real, reactive, and apparent powers in the system. Find the total current supplied to the distribution system by the utility.
FIGURE PA–4
The system in Problem A–6.

(b) Repeat part (a) with the switch closed.

(c) What happened to the total current supplied by the power system when the switch closed? Why?

REFERENCE